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PROCEEDINGS OF THE WIRELESS SECTION

OF THE INSTITUTION.

EDITED BY W. K. BRASHER, SECRETARY.

Vol. 15 MARCH, 1940. No. 43

(Issued two or three times yearly.)

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E. and F. N. SPON, LIMITED, 57 HAYMARKET, LONDON, S.W.1.
CHEMICAL PUBLISHING COMPANY OF N.Y. INC., 148 LAFAYETTE STREET, NEW YORK.

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PROCEEDINGS OF
THE WIRELESS SECTION OF
The Institution of Electrical Engineers
VOL. 15.

WIRELESS SECTION: CHAIRMAN'S ADDRESS[*]

By E. B. MOULLIN, M.A., Sc.D., Member.[†]

"THE MOLECULAR NATURE OF A DIELECTRIC"

(*Address received 7th December*, 1939.)

For this Chairman's Address I had planned to give a lecture on the nature of dielectrics and to enliven it with some models and demonstrations. I had hoped to amuse an audience for an hour by describing molecular models in a rather free-and-easy way. I dare say much of what I should have said would have been familiar to most electrical engineers, but if it had been new to some I think it would have opened for them a way of approach as fascinating as it was pictorial, as severely practical as it was eminently rational. Now that war has come the lecture with its lighter touches and asides cannot be given and the Address can only be printed in all its academic seriousness.

(1) DIELECTRIC CONSTANT AND ITS MOLECULAR INTERPRETATION

(a) A first rough model.

It has been known for the larger part of a couple of centuries that the capacitance of a condenser is increased considerably by filling its air space with such substances as paraffin oil, glass or indiarubber: this experience leads directly to a definition of "dielectric constant." We may be content to accept the fact and call it another unexplainable mystery of nature, or we may prefer to ease our memories and try to comprehend it in terms of the inverse square law of force. If we succeed we shall have only this difficult and, in a sense, improbable hypothesis as the whole basis of electrical science. If we can tolerate only the inverse square law, then we have to find some more of our "invention, electricity" in the insulator. It is as though the condenser plates were closer together than they really are, or as though there were some invisible pieces of conducting material embedded in the insulator. Such conducting material would, *ipso facto*, contain the needed electricity. Science has a title to our respect from its results and not from the hypotheses by which it arrives at these results: the hypotheses are always more or less laughable, and it may take long to stop laughing at a new one and see what it is capable of producing.

Instead of scorning the idea of conducting particles we will render them invisible by having them rather small and sparsely scattered, and as a first shot will simplify mathematical tedium by making them spherical. As we focus our attention on one of them we see a conducting sphere in a sensibly uniform electric field which is due jointly to the electricity on the condenser plates and the separation of electricity in all the other spheres. Here is an old problem, and we know that the separated or induced charges on the sphere endow it with an electric moment which is equal to the volume of the sphere multiplied by the uniform electric field in which it is placed. It is then a very simple matter to calculate the capacitance of a condenser having n such spheres scattered sparsely in every unit volume, and we find readily that the dielectric constant resulting from their presence in the air space is given by the equation

$$\kappa^{\ddagger} = \frac{1 + \text{(twice the total volume of spheres)}}{1 - \text{(the total volume of spheres)}} \quad . \quad (1)$$

Equation (1) shows that the net result does not depend on the radius of the spheres, which need not, therefore, be all equal: the analysis, however, does rely on the total volume being much less than unity. We can now say that if some material has a dielectric constant of 2 it is as though one-third of it consisted of conducting spheres. Obviously this is not true, at any rate in the sense in which we have pictured it. Let us see if perhaps the trouble is not that the picture was essentially impossible but that it was too naïve.

---

[*] Reprinted from *Journal I.E.E.*, 1940, **86**, p. 113.

[†] University of Oxford.

[‡] See List of Symbols on page 15, at end.

(b) A second model, this time molecular.

We have been driven to realize that our invented stuff, electricity, cannot be subdivided indefinitely, and have had to cumber the first simple hypothesis by making it consist of identical fragments called electrons. This concession has paved the way to the picture that molecules differ in kind largely because they contain a different number of electrons disposed like planets round a positive charge, called a proton, as sun. In Fig. 1 we draw a naïve picture of such an atom (hydrogen). Here P is the proton and e the electron in a circular orbit of radius a, the plane of the ecliptic being perpendicular to an electric field ϵ. The electron attracts the proton with a force e^2/a^2, and vice versa. What maintains the radius a or the perpendicularity of the ecliptic to the electric field we refuse to inquire here. The electric field exerts an upward force $e\epsilon$ on the proton and a like downward force on the electron, and this must cause the proton to move away from the centre of the circular orbit. If it moves a distance x so small that the distance between the two charges is increased insensibly, the force between them will be unchanged but will now have a component perpendicular to the orbit equal to

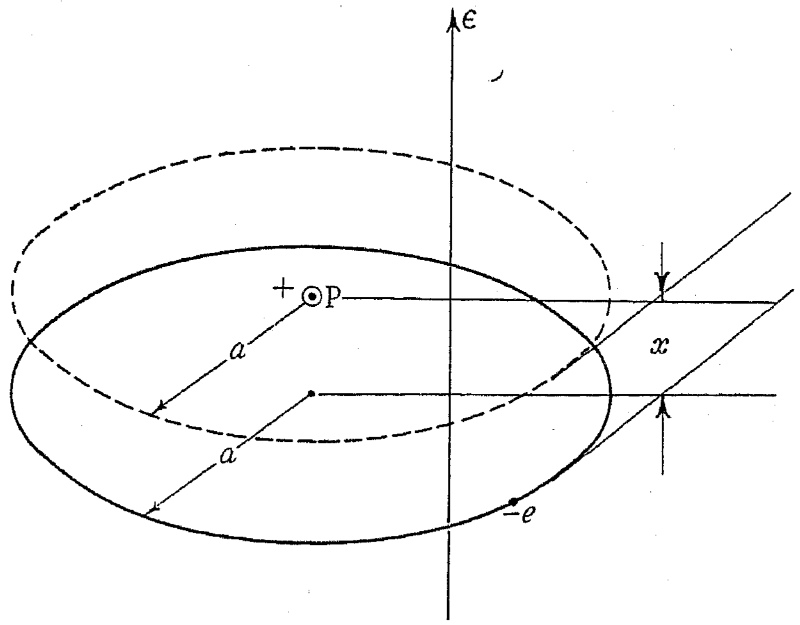

Fig. 1.—Distortion of hydrogen atom in an electric field.

$$\frac{e^2}{a^2} \times \frac{x}{a}$$

which is equilibrated by the external stretching force $e\epsilon$ provided by the superposed field. The atom now has an electric moment in the direction of the applied field, and its value is

$$M = ex = a^3\epsilon \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Had the atom been a conducting sphere of radius a, it would also have had a moment of the same magnitude. Hence equation (1) would still be valid, though now we should write total volume of molecules vice total volume of spheres.

Without, however, relying on the direct observations of X-ray crystallography, of which we shall speak later, we are able to deduce the volume of a molecule by interpreting certain simple measurements in terms of the kinetic theory of gases. Hence we are able to test the enlarged interpretation of equation (1), and can compare the molecular volume as deduced from measurements of dielectric constant with the molecular volumes as deduced from measurements which are in no way electrical: let this ratio be called z, where z would be unity if the model were perfect. The values of z for four very different molecules are shown in Table 1. The mean value is 0·599: so close an agreement is remarkable and encouraging.

A similar table for 28 ions of inert-gas structure has been compiled by F. C. Frank,[1] and in this the mean value of x is 0·701: the range is from 0·166 to 1·5, with seven cases where the value is greater than unity. Thus we see that the dominant factor in determining the dielectric constant of a substance is what we engineers would call the " space factor " of the material packed between the condenser plates: another factor is the z value of that material. Recent years have seen the introduction of ceramic-type materials having large dielectric constants. One well known material is composed largely of titanium dioxide. Now the value of z for the titanium ion is 1·04 and for the O_2 ion is 1·2, and hence the molecule is inherently capable of giving a dielectric constant larger than most. The material is polymorphic and exists in three forms: the importance of space factor with a given molecule is well illustrated by Table 2.*

Table 1

Molecule	z
H_2	0·488
A	0·52
C_2H_4	0·67
C_5H_{12}	0·72

The model pictured in Fig. 1 operates by virtue of the proton being strained by the field out of the plane of the electron orbit: the action depends essentially on the valency electrons, and the mechanism is often called electronic polarization and will occur in each atom of a polyatomic molecule. We picture chemical union as due to exchange or sharing of these valency electrons between the two or more atoms of a molecule, and such exchange causes each component atom to be charged or ionized: it is, however, exchange and not robbery, so that the molecule is not charged as a whole any more than its component atoms were charged before the union occurred, though they may be said to be charged after the union. The electrical centres of such a molecule may be strained apart by an electric field, and this would give rise to a polarization which is additional to the electronic polarization of the dismembered atoms. Such mechanism is called atomic polarization: it involves bulk movement of

Table 2

	Rutile	Brookite	Anatase
Density	4·21	4·11	3·87
Dielectric constant . .	114	87	48

---

* In this connection, pages 517 and 518 of Reference (1) should be consulted.

heavy atoms in contrast with movement of electrons within the atom. The first may be likened to two large masses connected by a spring, and the second to a very small mass connected (by a comparable spring) to another which is some 2 000 times greater. The natural frequency of the first will be vastly less than that of the second, and hence the practical possibility of an approach to resonance with the first is much greater than with the second. If the dielectric constant of a given material has an appreciable contribution from atomic polarization, we must be prepared to find the dielectric constant dependent on frequency in the infra-red region of the frequency spectrum.

(c) Effect of temperature.

All molecules are agitated by a ceaseless chaotic motion, the energy of which is familiar to us as temperature. In the gaseous state the molecules, spinning violently, travel many diameters between collisions. In the liquid state the packing is so close that the free path is only of the order of a diameter, and the spin can seldom turn the molecule through a complete revolution between successive collisions: on the average it will take hours for a given molecule to move its average position by, say, 1 mm. or to turn its average direction through a complete revolution. In the solid state the molecules have an assigned place in the crystal lattice and vibrate and twist about this mean position. But whatever the state, each molecule will on the average have kinetic energy $\frac{1}{2}kT$ in every degree of freedom possible to it: here k is the same for all molecules (it is Boltzmann's constant and equal to $1 \cdot 37 \times 10^{-16}$ erg) and T is the absolute temperature. Thermal agitation and the principle of equipartition of energy has forced itself loudly on the attention of radio engineers; for it is responsible for the background noise in circuits and sets a limit to the degree of amplification which can be used with benefit. I and my associates have filled so many pages of the *Journal* with discussion of thermal agitation that I need not trespass further on its space: let this introduction suffice to remind the reader of the mechanism which we must think of once more if we are to understand dielectrics.

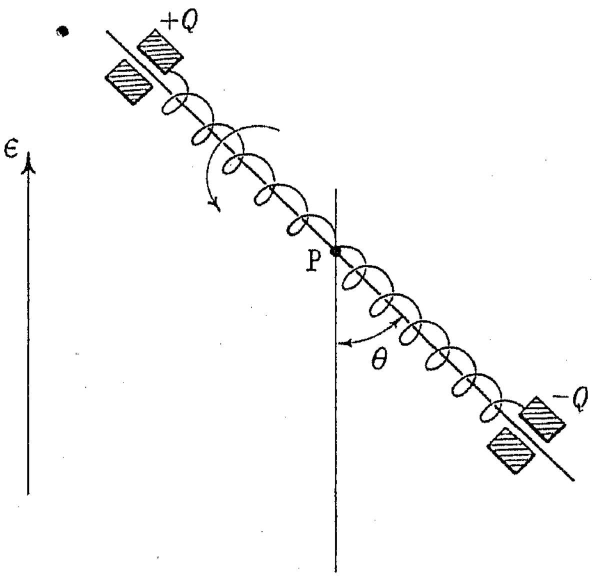

Fig. 2

Thus we now admit that the model atom depicted in Fig. 1 is moving as a whole and also twisting round in the field. Will this affect the result? It will not affect it, because the controlling force is proportional to the distance through which the proton moves away from the ecliptic plane: the time average of the moment will not be altered by the heat motion. For the gaseous state we may examine the problem in terms of classical mechanics very simply: it is the same as examining the motion of the mechanism shown diagrammatically in Fig. 2. Two massless charges Q can slide freely on a rod fixed perpendicular to a horizontal axle P which turns in frictionless bearings: Q is attached to P by a helical spring. The system is set rotating and then left free in an electric field ϵ perpendicular to the axle P. During each revolution the electric field will cause the charges to move to and fro along their guide rods. The electric moment will be $\alpha\epsilon \cos \theta$, where α is the spring constant and the component moment in the direction of the field will be $\alpha\epsilon \cos^2 \theta$. This pulsating moment will cause the angular speed to hunt about its mean value: it is necessary to find the time average of $\cos^2 \theta$ during a whole revolution. It can be shown that:—

$$\frac{\alpha\epsilon \int_0^t \cos^2 \theta dt}{t} = \frac{\alpha\epsilon}{2}\left(1 + \frac{3}{8}y^2\right)$$

where t is the time of a revolution and $y \equiv \alpha\epsilon/(2kT)$. We can soon show that y is a very small quantity. For, according to Fig. 1, $\alpha = a^3$, and for all atoms a is of the order of 10^{-8} cm. (1 Ångström unit): thus if ϵ is 3 kV per cm. (10 e.s.u.) then $\alpha\epsilon$ is of the order of 10^{-23}. If T is 300° abs., $kT = 4 \cdot 1 \times 10^{-14}$, and thus y^2 is of the order 10^{-20}.

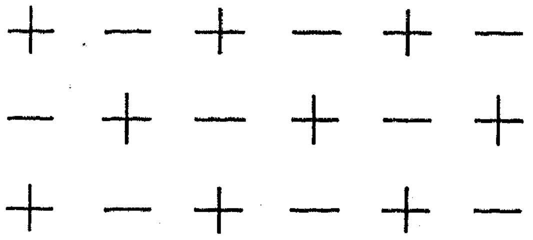

Fig. 3

It is of fundamental importance to realize that electron polarization is not affected by temperature, and hence it follows that the temperature coefficient of dielectric constant, a parameter vitally important to radio engineering, is due only to change of density and is therefore essentially negative. If it is desired to have a positive or zero temperature-coefficient, it is useless to employ a material which responds only to electron polarization. It follows readily that, for a material of density ρ,

$$\text{Temperature coefficient of } \kappa \equiv \frac{1}{\kappa}\left(\frac{\partial\kappa}{\partial T}\right)_P = \frac{(\kappa - 1)(\kappa + 2)}{3\rho\kappa}\frac{\partial\rho}{\partial T} \quad . \quad . \quad (3)$$

and is negative since $\partial\rho/\partial T$ is negative: also $\frac{1}{3}\frac{\partial\rho}{\partial T}$ is equal to the temperature coefficient of linear expansion. The atomic polarization in the molecule itself is very small compared with the electronic polarization in the individual atoms, and is consequently difficult to determine: for example, for ethylene (C_2H_4) the ratio is about $4\frac{1}{4}$ % and for normal pentane (C_5H_{12}) it is about $2\frac{1}{4}$ %. It is believed [see, for example, Reference (2)] that the atomic polarization, like the electronic, is unaffected by change of temperature or change of state. However, when ionized molecules are arranged in a solid crystal lattice, as indicated diagrammatically in Fig. 3, they contribute an atomic polarization which is subject to temperature variation. A rise of temperature expands the lattice: an increase of distance between the ions tends to decrease the attractive forces between them and thus makes more pronounced a concertina action when the whole aggregate is placed in an alternating electric field. Thus the contribution to dielectric constant from such a mechanism will have a positive temperature-coefficient. We are familiar with ceramic materials which have positive temperature-coefficients and with others in which this coefficient is negative, and we know that the two are sometimes combined in small trimming condensers whose temperature coefficient is adjustable through zero. I suspect such ceramics act by virtue of a nicely selected mixture of materials, one of which exhibits pronounced atomic polarization. Frank[1] cites calcium fluoride as an interesting example of the opposed tendencies of electron and atom polarization in respect of temperature-changes: the coefficient due to the density change is $-0\cdot 57 \times 10^{-4}$ and the estimated coefficient due to loosening of the crystal is $+1\cdot 37 \times 10^{-4}$, giving a resultant of $+0\cdot 87 \times 10^{-4}$ which in fact is much less than the observed positive coefficient.

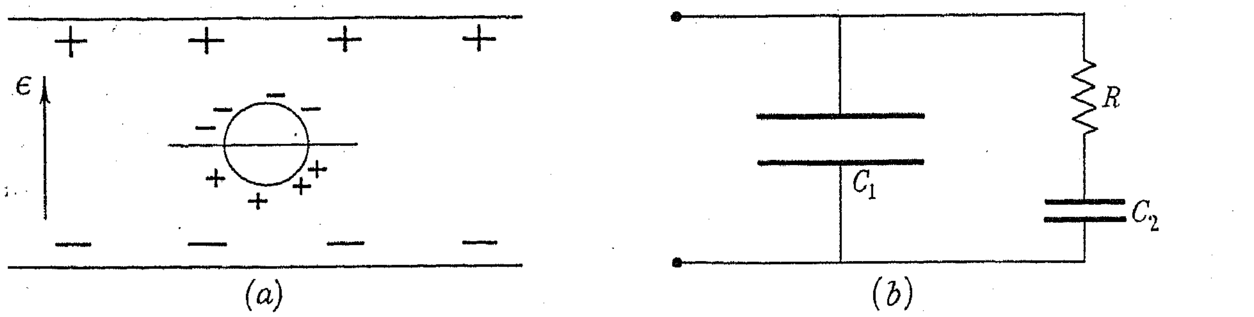

Fig. 4.—Conducting sphere between the plates of a condenser, and equivalent circuit diagram.

Having attuned our minds to the concept of the forces within the atoms and the distortions which provide the dielectric constant, we will make an estimate of their order of magnitude. The electron charge is $4\cdot 8 \times 10^{-10}$ e.s.u. and the radius of all atoms is of the order of 10^{-8} cm.: hence the field at an atomic radius from an electron is of the order of 15 000 megavolts per cm., in comparison with which the fields we can superpose sink into insignificance. Again, suppose that Fig. 1 represents a proton and a valency electron separated by one Ångström unit (10^{-8} cm.); then, from equation (1),

$$\frac{x}{a} = \frac{a^2\epsilon}{e} \simeq \frac{2}{10^6}$$

if ϵ is equal to 3 kV per cm. Thus the distortions which we produce, and which do in fact provide the dielectric constant, are incredibly small in fractional amount. Though the charge on a single electron is so small as to seem to some of us of little account as such, it must not be forgotten that we are concerned with distances whose inverse square is very large. A healthy respect for electronic forces will perhaps be engendered by a numerical example, due to Debye: A gramme-atom of sodium ions (23 g.) at the north pole of the earth would attract a gramme-atom of chlorine ions (35·5 g.) at the south pole with a force equal to the weight of 52 tons. At any given distance the electrical attraction between a sodium ion and a chlorine ion is 10^{33} times the gravitational, and thus it follows that it is the electrical forces alone which matter appreciably.

(d) Energy loss in dielectrics.

Energy loss in dielectrics is a familiar and distressing experience. Some will have encountered it through the attenuation it produces in telephone cables or by the limit it sets to the selectivity of circuits, and others by the melting of high-frequency insulators. Is there any mechanism for this loss in the atomic picture given so far? The answer is " No." Movements of the valency electrons in the atoms cannot cause an absorption of energy in the atom as such, nor can they increase its kinetic energy (save at a resonance which can occur only in the far ultra-violet), which would in time be transferred by collision to neighbouring atoms and be disclosed as a rise of temperature of the whole. In a sense there ought to be no such thing as dielectric loss and in one view of experience there is indeed precious little. Thus good lenses absorb very little light, and the damping of a quartz piezo-electric crystal is exceedingly small (hence its valuable properties) even when accompanied by mechanical concertina action. Indeed, it seems certain there is no dielectric loss, barring perhaps sundry narrow absorption bands in the infra-red, in pure and regularly built crystals.[3] The loss we experience is often due to conducting impurities or to stray ions which move as a whole in the field and transfer the kinetic energy they acquire to the surrounding material. It is instructive to examine the mechanism of loss due to the presence of conducting particles.

Thus imagine a solid sphere of conducting material between the plates of a condenser [Fig. 4(*a*)]. Negative charge will be induced over the upper hemisphere and positive over the lower, having a magnitude and distribution such that it produces inside the sphere a uniform electric field equal and opposite to the field ϵ due to the charged condenser plates; the resultant internal field being precisely zero. If the charge on the plates is alternating there will be a flow of electric current inside the sphere, the lines of flow being parallel to the applied field [i.e. vertical in Fig. 4(*a*)]. Since current is the rate of change of charge it follows that the current must increase in proportion to the frequency if the induced charge on the hemispheres is to equal the value it would have in the instantaneous value of field, if static: thus the energy loss tends to increase as the square of the frequency. The resultant field inside the sphere cannot, however, remain zero with increasing frequency, but must equal the field necessary to drive the current through the resistance of the sphere. Hence the charges induced on the hemispheres will decrease with increasing frequency and thus the current will not increase so rapidly as we supposed at first, since the net internal field is equal to the current density multiplied by the specific resistance: thus the current density cannot exceed the value ϵ/ρ and the energy loss must approach a constant value when the frequency becomes very high. Clearly, then, there is some frequency at which the loss is a maximum. At very low frequencies the sphere increases the capacitance of the condenser because it fills up some of the space between the plates: at very high frequencies the net field inside the sphere tends to the value ϵ and the sphere might as well be absent in respect of its ability to increase the capacitance of the condenser. It is almost obvious that Fig. 4(*a*) can be redrawn in terms of familiar circuit elements as Fig. 4(*b*), and it can be shown that the lead angle of this combination is less than $\pi/2$ by the angle δ, given by

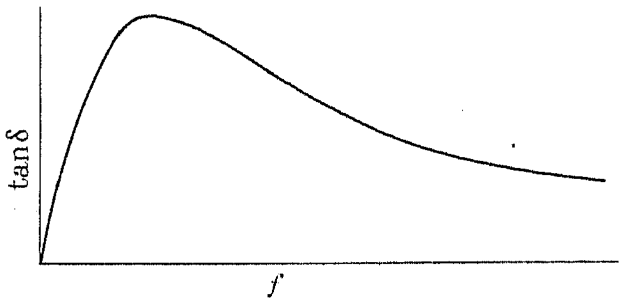

Fig. 5

$$\tan \delta = \frac{RK}{1 + R^2p^2K^2\dfrac{C_1 + C_2}{C_1}}$$

where $$K \equiv \frac{C_1C_2}{C_1 + C_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and $$p = 2\pi \times \text{frequency.}$$

It follows that $\tan \delta$ is a maximum when

$$RpK = \sqrt{\left(\frac{C_1}{C_1 + C_2}\right)} \quad . \quad . \quad . \quad . \quad (5)$$

and hence

$$(\tan \delta)_{max} = \frac{RK}{2}$$

It is obvious from Fig. 4(*b*) that for very low frequencies the effective capacitance is sensibly equal to $(C_1 + C_2)$ and for very high frequencies sensibly equal to C_1. The sphere in Fig. 4(*a*) is intended to represent one of many small pieces of conducting impurity or droplets of moisture: the only case of practical interest is when their total volume is small, and then C_2/C_1 is much less than unity, and (5) may be written

$$RpC_2 \simeq 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Consideration, however, will show the reader that for a sphere R increases in proportion to the radius, while C_2 varies inversely as the radius, and hence RC_2 is a constant depending only on the specific resistance. Less commonplace analysis shows that (6) should then be replaced by

$$\frac{3}{4\pi}p\rho = 1$$

that is $$f = \frac{2}{3\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

Thus the condition for maximum $\tan \delta$ depends neither on the radii of the spheres nor on their total number: the maximum value of $\tan \delta$ is proportional to C_2 and thus to the total number of spheres, or, in other words, to the fractional amount of moisture or impurity present in the dielectric. The resistivity of ordinary tap water at room temperature is about 10^5 ohm-cm. units, and for this $\tan \delta$ would be a maximum at a frequency of 6 Mc./sec.

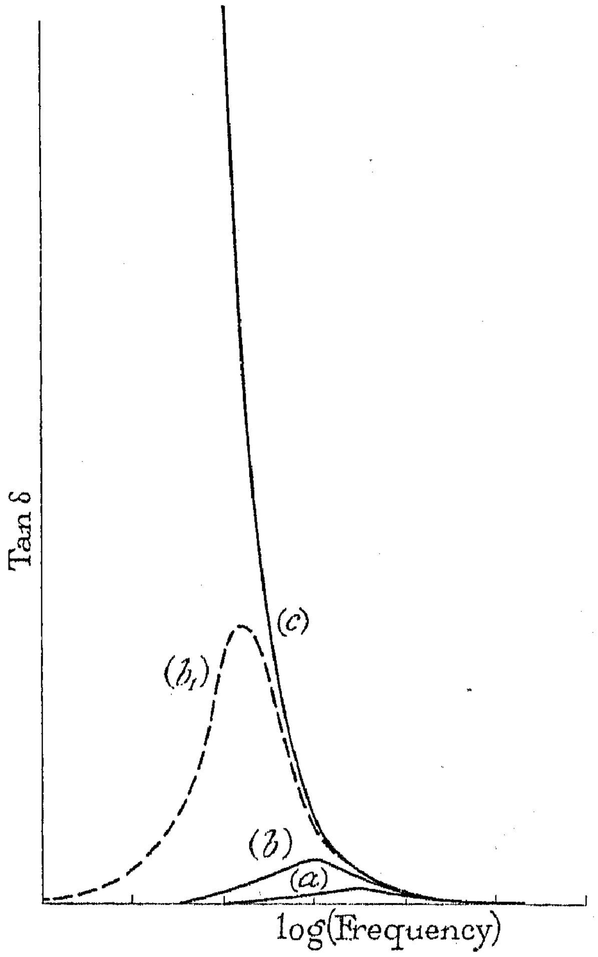

Fig. 6.—"Loss angle" caused by a given quantity of conducting material in the form of (*a*) sheet, (*b*) spheres, (*c*) cylinders.

I have now given in brief outline the classic analysis of K. W. Wagner. According to it, the curve relating $\tan \delta$ and frequency (expressed in terms of that which makes $\tan \delta$ a maximum) is shown in Fig. 5, and this needs but a glance to tell us that it is quite contrary to all ordinary experience.

The cause of the discrepancy is the assumption that the droplets of moisture are spherical, and it is easy to see that the behaviour of the dielectric must be very sensitive to the shape of the particles. For if the particles become thin needles they may ultimately form a conducting bridge between the condenser plates; the resistance of this shunt being equal to $\rho d/v$, where v is the fractional volume of moisture in the dielectric and d is the distance between the condenser plates. Then it follows readily that

$$\tan \delta = \frac{2v}{\rho f}$$

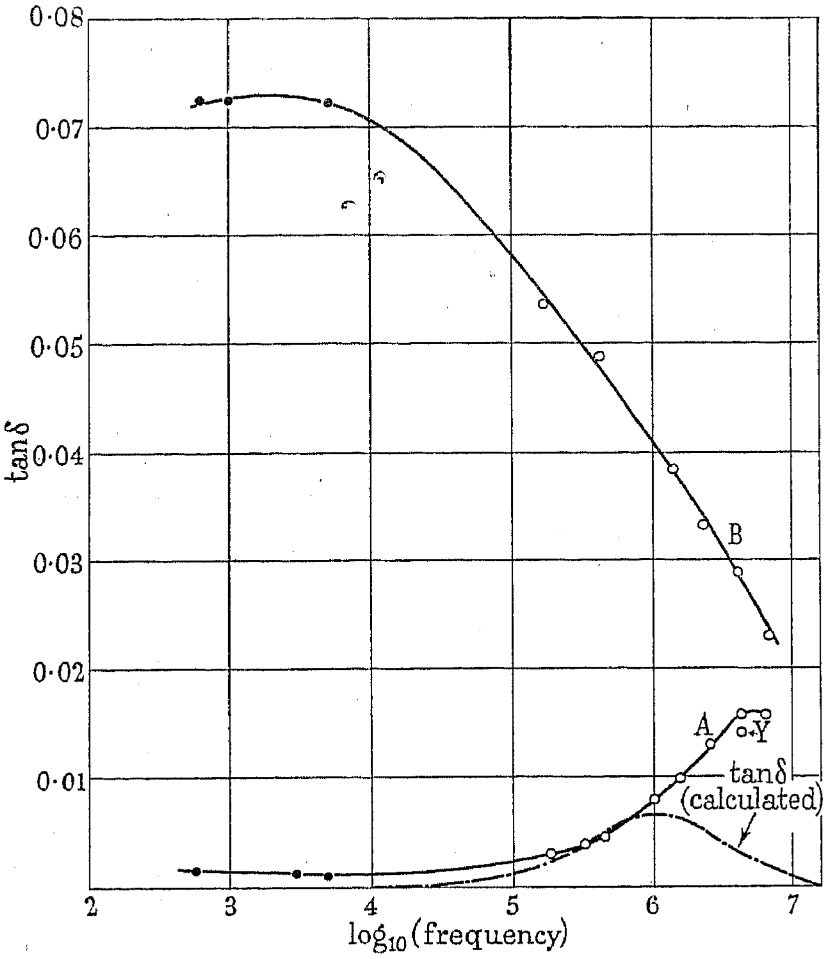

Fig. 7

Curve A. 9 deg. C. below solidifying point.
Curve B. Room temperature.

Thus with such a shape of particle the maximum corresponding to that shown in Fig. 5 would be at zero frequency, and then its value would tend to infinity. The other extreme is to suppose the moisture is spread in a uniform sheet parallel to the plates: then it can be shown that the maximum value of $\tan \delta$ occurs when $f = 2/\rho$ vice $f = 2/(3\rho)$ for spheres: also $\tan \delta_{max} = v/2$ vice $3v/2$ for spheres. The relation between $\tan \delta$ and frequency is shown for these three shapes of particle by the three full-line curves in Fig. 6. It is now clear that a given amount of moisture in a dielectric is capable of producing almost any power factor depending on the shape of the particles: the power factor may rise to a maximum at any frequency between zero and $2/\rho$. Since the droplets of moisture are likely to assume a variety of shapes, the power factor/frequency curve of a practical dielectric is likely to be the resultant of many curves typified by Curves (*a*), (*b*) and (b_1) in Fig. 6, and hence any maximum is likely to be very blunt: this is in accordance with experience. The effect has been shown experimentally in a very beautiful manner by R. W. Sillars, who produced a moisture-laden dielectric by shaking up 25 g. of water in 300 g. of molten paraffin wax. His idea was that the droplets would be spherical in the molten wax and remain approximately spherical so long as the wax was hot enough to be soft; but that when the wax hardened and contracted, minute internal cracks would form into which the moisture would run and thereby alter enormously the particle shape. The variation of $\tan \delta$ with frequency was observed both when the wax was maintained at 9 deg. C. below its setting point (i.e. at about 47° C.) and when it had been allowed to cool to room temperature. The results obtained are shown respectively by Curve A and Curve B in Fig. 7, and the contrast is as instructive as it is striking. Thus at 100 kc./sec. the power factor due to 6·7 % of water was about 0·2 % and increasing with frequency when the droplets were roughly spherical: when the wax had hardened, the power factor at this frequency had risen to 5·5 % and was decreasing with frequency. In the first case the effect of the moisture was almost insignificant, whereas in the second case the dielectric would have been useless in practice. Could there be more convincing proof than Fig. 7 of the fact that a mere statement of moisture content in a dielectric can be no guide to the electrical behaviour of a dielectric?

(2) POLAR MOLECULES AND THE EFFECT OF THEIR PRESENCE IN A DIELECTRIC

All molecules are electrically neutral, but it does not follow from this that the positive and negative charges are distributed with sufficient symmetry to give no external electric field. Thus, consider a simple molecule such as water, H_2O: are the two positive hydrogen ions arranged as in (*a*) or (*b*) or (*c*) of Fig. 8?

Arrangement (*a*) would have no external moment and the force would vary as the inverse fourth power of the distance. Arrangements (*b*) and (*c*) would clearly have a net moment and the force would vary as the inverse cube of the distance. It can be shown that Arrangement (*b*) is essentially unstable and hence must be ruled out: also that Arrangement (*c*) is stable only if θ is 32° or 55°.

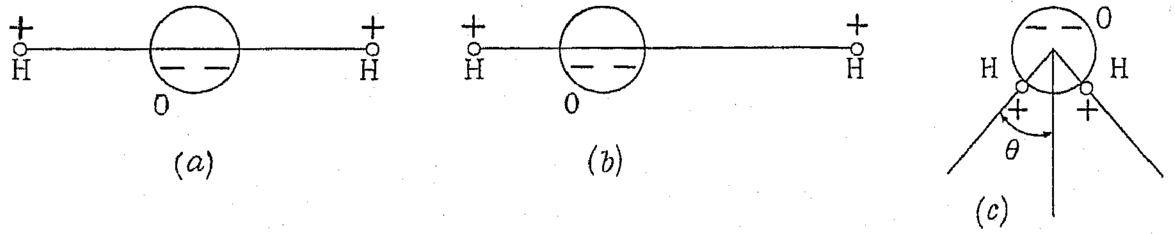

Fig. 8.—Linear and triangular H_2O models.

The problem of whether or not any particular molecule has an inherent electric moment can be tested experimentally by firing a beam of molecules, reminiscent of the electron beam in a cathode-ray tube, through a non-uniform electric field. It is not difficult to see that the deposit pattern on the target will be modified by the electric field only if the neutral molecules are inherently polar: further, it must be possible to deduce the magnitude of the moment from the shape of the deposit pattern. Such experiments have shown[4] that some molecules are polar and some are not. Water (H_2O), hydrochloric acid (HCl), ammonia (NH_3) and ethyl alcohol (C_2H_5OH) are examples of polar molecules; while nitrogen (N_2), methane (CH_4), carbon tetrachloride (CCl_4) and pentane (C_5H_{12}) are examples of non-polar molecules. All polar molecules have an inherent electric moment (μ) whose value is of the order of that due to a pair of electronic charges separated by the atomic radius, namely $\frac{4\cdot 77}{10^{10}} \times \frac{1}{10^8}$; thus the moment of nitrobenzene is $3\cdot 9 \times 10^{-18}$ and of water is $1\cdot 51 \times 10^{-18}$. Referring now to Fig. (8*c*), it can be shown that if $\theta = 32°$ then $\mu = 1\cdot 34 \times 10^{-18}$, whereas if $\theta = 55°$ then $\mu = 4\cdot 32 \times 10^{-18}$. Thus it is concluded that the smaller of the two possible values of θ is the true one. This is a fascinating example of the way in which the structure of the molecule itself can be deduced, but the reader may well ask what this has to do with dielectrics and with radio communications.

If the molecules of water are inherently polar, surely they will try to set themselves parallel to an electric field (like the corresponding behaviour of the supposed magnetic particles of iron) and endow water with a very large dielectric constant. It is well known that the dielectric constant of water is about 80, in striking contrast with a value of 2 or 3 for most other materials. Here then we have at last a rational explanation of this outstanding exception among common materials. Let it not be forgotten, however, that water has a dielectric constant of 80 at all radio frequencies, but that this falls to about 2 for light waves. Familiar experience suggests at once that the polar orientation becomes ineffective at very high frequencies and then there remains only the effect due to straining the shape of the molecule (valency electrons) discussed fully in the opening sections of this Address.

Let us now try to picture how a dipole can orientate itself in the direction of an applied field. First of all it is certain that the orientation cannot be complete, for, if it were, the dielectric constant would tend to infinity and there would be marked saturation effects, reminiscent of magnetized iron: both these things are contrary to experience. First consider polar molecules in a gaseous state and remember that now the medium consists of widely separated particles, all moving hither and thither with velocities of the order of that of sound, colliding with one another and with the walls of the container, producing on it the effect we call pressure. Also the collisions cause them to spin and twist while performing their straight-line paths between collisions. Though every conceivable velocity of translation and rotation is to be found, yet there is one which is vastly more probable than all others. The average translational energy (or mean square velocity) depends only on the temperature and not on the kind of molecule or on the molecular density. The average translational energy is $\frac{3}{2}kT$, where k is Boltzmann's constant ($1\cdot 37 \times 10^{-16}$ erg) and T is the absolute temperature. On the average the energy of rotation bears a definite relation to the energy of translation and is equal to $\frac{1}{2}kT$ for rotation about each of the three principal axes of inertia: commonly this is expressed by the statement that on the average there is energy $\frac{1}{2}kT$ in every mode of motion.

Now any orientation effect must occur while the molecules are free between collisions, and, further, the time they are in contact is negligible. Evidently the twisting effect of the field on the dipole will be resisted by the gyrostatic effect of its spin, and the net result is likely to be small. To understand the mechanics of the effect, consider the following simple problem, which we will state first in terms of a mechanism such as an engine governor.

Let the light rod AB (Fig. 9), mounted vertically in frictionless bearings, have attached to it by a smooth hinge at B a light rod BC of length l to which is attached at C a small mass m. Let there be some device (such as a peg in a hole) which is capable of holding BC perpendicular to AB. Let the shaft AB be spun up to an angular velocity ω and then left free: it will continue to rotate at ω indefinitely, since the bearings are frictionless. At any time ($t = 0$) let the peg be withdrawn so that C starts to fall. CB will then oscillate for ever, and θ will change periodically from zero to a maximum value. It follows from simple dynamics that

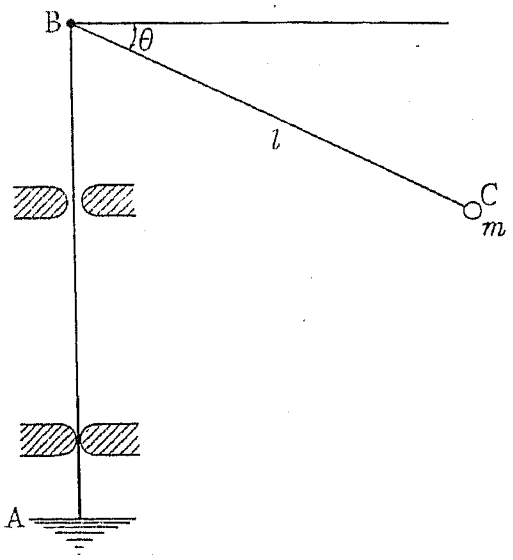

Fig. 9

$$\dot{\theta}^2 = \omega^2(s \sin\theta - \tan^2\theta)$$

where

$$s \equiv \frac{mgl}{\frac{1}{2}ml^2\omega^2}$$

Hence it follows that $\theta = 0$ when

$$\sin\theta = \frac{\sqrt{(1 + 4s^2)} - 1}{2s} \simeq s, \text{ if } s << 1.$$

Hence if s is small, θ is always small, and we may write

$$\dot{\theta}^2 \simeq \omega^2(s\theta - \theta^2)$$

$$\therefore \quad \ddot{\theta} = \frac{\omega^2}{2}(s - 2\theta)$$

Hence it follows that BC will oscillate about its mean position ($\theta = s/2$) with frequency $\omega/(2\pi)$. Now suppose that BC is extended a distance l in the direction CB and that the single mass m in a gravitational field g is replaced by equal and opposite charges e (at C and C′) in an electric field ϵ. The motion will be unchanged, but now we must write

$$s \equiv \frac{e\epsilon l}{\frac{1}{2}ml^2\omega^2} \equiv \frac{2e\epsilon l}{kT} \equiv \frac{2\epsilon\mu}{kT} \quad . \quad . \quad . \quad (7)$$

Thus the applied field causes the plane of rotation to oscillate and in consequence the rotating dipole has a very small component moment in the direction of the field: the component fluctuates between zero and a maximum each revolution, but its average value is

$$\overline{M} = \frac{\mu s}{2} = \frac{\mu^2\epsilon}{kT}$$

in the direction of the field.

Now consider a dipole which is revolving about an axis P perpendicular to the field, as indicated in Fig. 10. While θ increases from 0 to π the sense of the couple due to the field is such as to assist rotation: as θ increases from π to 2π it impedes rotation. Hence the angular velocity, when in the applied field, is not constant but hunts above and below the mean value ω: it is a minimum when $\theta = 0$ in Fig. 10. Hence the positive charge spends more time below the pivot P than it spends above it, and consequently the time average of the moment in the direction of the field is not zero, but has a small value *downwards*. Treatment analogous to that used previously shows that now

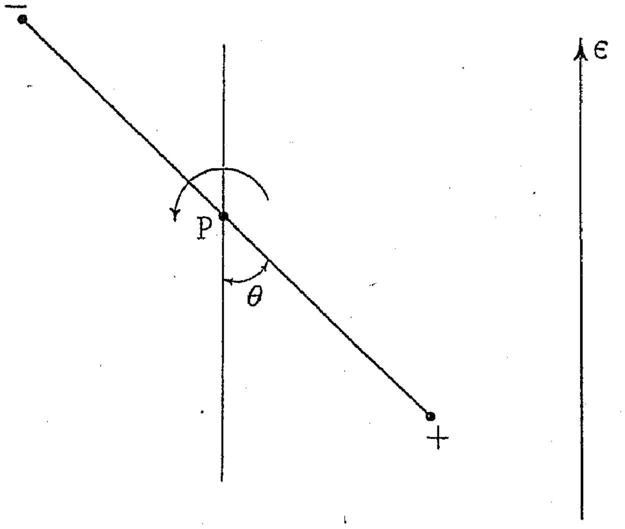

Fig. 10

$$\overline{M} = -\frac{\mu^2\epsilon}{2kT}$$

the negative sign meaning that the direction is against the field.

In a representative sample of molecules it may be supposed that one-third are spinning about an axis parallel to the field, and one-third about each of two mutually perpendicular axes. Hence the net moment for N molecules would be

$$\overline{M}_N = \frac{\mu^2\epsilon}{kT}\left[+\frac{N}{3} - \left(\frac{N}{3} \times \frac{1}{2}\right) - \left(\frac{N}{3} \times \frac{1}{2}\right)\right] = 0$$

Thus arises the paradox, which I have never resolved to my satisfaction, that while every molecule is affected by the field the net result is zero.

This, however, is certainly contrary to experience, for a gas of polar molecules has a dielectric constant very different from that of a non-polar gas: moreover, it is contrary to the result of a calculation made according to the most general principles of statistical mechanics and having no reference to a particular mechanism. There is a fundamental proposition known as the Maxwell–Boltzmann distribution theorem which is applicable to any aggregation of molecules which are in thermal equilibrium and which are in a field of force. Thus this general theorem tells us at once that the distribution of particles in an isothermal atmosphere must be exponential from the ground upwards: this involves no use of the gas laws. When this theorem is applied to polar molecules in an electric field, it shows that the result will be as though each dipole were at rest and had a moment $\overline{M} = \mu^2\epsilon/(3kT)$ in the direction of the field. Hence by some means or other they do not neutralize completely. Where is the mistake? Is it in the simplification introduced in the analysis by presuming that

$$\tfrac{1}{2}kT \equiv \tfrac{1}{2}\mathrm{I}\omega^2 >> \mu\epsilon$$

Taking $\mu = 1\cdot5 \times 10^{-18}$ and $\epsilon = 3$ kV per cm., then $\mu\epsilon = 1\cdot5 \times 10^{-17}$; if $T = 300°$ abs., $kT = 4\cdot11 \times 10^{-14}$: hence $\mu\epsilon/(3kT) \simeq 1/10^4$, and thus is very small compared with unity.

Apparently the mistake has come about by assuming that all molecules are spinning with the same energy, whereas in fact all spin energies, from zero upwards, are represented. Thus it seems that the predominant contribution is provided by those molecules which have very little spin. All are affected by the field: all those whose spin is near the average value mutually neutralize one another and might as well not be present; those which so to speak "do the trick" are those nearly at rest. I suspect it would help our knowledge of how to make good dielectrics if our minds could visualize step by step a vast series of collisions and see that it is only the unenergetic ones which matter. However, what I will call the paradox of the Boltzmann theorem is in fact familiar to all of us if we stop to think. Let us compare a gas molecule in the atmosphere to a perfectly elastic tennis ball, and let the atmosphere be so rarefied that collisions do not occur. The ball will descend from the skies, hit the ground with the velocity appropriate to the temperature, rebound and ascend once more to a certain height, coming very slowly to rest, for a moment, before descending again. It will spend most of its time high up in the atmosphere, and in such circumstances the atmospheric density will increase from the ground upwards. In such a world an airman and his engine would experience breathing difficulties only when low down. Somehow the collisions and the resulting distribution of velocities make things happen as we know them and as Boltzmann statistics predict, but do any of us really see that it just could not be otherwise? Again, suppose that on the ground there is a container full of hydrogen gas at atmospheric pressure and temperature: if the cover is removed the hydrogen will ascend and distribute itself with a density decreasing exponentially from the ground upwards. Somehow the centre of gravity of the hydrogen is raised enormously. If we liken the air and the hydrogen to agitated tennis and ping-pong balls respectively, is it obvious that the ping-pong balls must on the whole be driven upwards by collisions with the tennis balls? It is puzzles like these which make it hard to think clearly about the behaviour of dielectrics.

Be this as it may, however, it is quite certain that if a molecule has an inherent moment μ then equation (2) will have to be written as

$$M = \left(a^3 + \frac{\mu^2}{3kT}\right)\epsilon$$

and hence it follows that the dielectric constant will decrease as the temperature rises. This effect is well known in gases and liquids which are polar, such as ammonia or nitrobenzene. Indeed, the effect is used as the ordinary method of measuring the dipole moment μ, since at present the precision of the dielectric-constant method much exceeds that of the more direct molecular-beam method.

A polarizability depending on temperature is in very marked contrast to all that was said in Section 1(*c*), and the reader should be careful to note that what was said there in respect of temperature coefficient is valid only if the dielectric is not inherently polar.

We have seen that $\mu\epsilon/(3kT)$ is of the order of 10^{-4}, and thus we can if we like say that on the average every molecule contributes about 1 part in 10^4 of the moment it is endowed with inherently. It is tempting to use a simplified and, I suspect, misleading picture in which the molecules have no spin energy and are free to turn themselves along the field but are continually being knocked out of alignment by the thermal collisions of translatory movement. Thus suppose a person in an agitated and closely-packed crowd were determined to keep his eye directed at a mark on the wall: bumps from his neighbour would largely frustrate his aim, but not completely so. Such a model is easy to visualize, and I suspect is tacit in much that has been written on this subject: it must be wrong and I suspect it is misleading. But at least so far as the net effect is concerned it is permissible to say it is as though the molecules had no spin and no inertia and were distributed with directions at random, and each were acted on by a restraining couple equal in magnitude to $2kT$.* You may put it which way you like—inertia and no restraining force, or no inertia but restraining force present. These are two alternative methods of stating the same result.

What will happen when the applied electric field is alternating? There must be a time effect in the Boltzmann distribution law, which can surely be true only after an infinite time for adjustment. Accordingly it will surely take time for the result $\mu^2\epsilon/(3kT)$ to come to pass, and hence it will never be fulfilled accurately in an alternating field. As a guess, if the alternation becomes very rapid there will be no time to reach even approximately the Boltzmann distribution, and thus the dielectric constant will decrease with increasing frequency. This we know is the case with water, where the dielectric constant falls from, say, 80 to 2 as the frequency increases from the infra-red to blue light.

The treatment of polar molecules in an alternating field is due almost entirely to P. Debye. Before giving his treatment, we will look at the problem by analogy. Gas in a container is in the gravitational field and, as we know, the density distribution is not uniform; though in a small container we do not usually bother to take count of the difference of barometer reading between the top and the bottom. Suppose now that in some unspecified manner the gravitational field is screened off: a finite time will be required for the density to become uniform by diffusion. At any given point the density will change exponentially with time and the initial state will not be *relaxed* instantly. Just as we talk of the time-constant of an *RC* circuit we might talk of the relaxation time of the gas in the container: it would depend on the gas and not on the size of the container.

It is a problem of diffusion and would involve the agitation velocity and mean free path in the same manner as these are involved in the coefficient of viscosity. Thus it would be permissible to express the relaxation time of the gas in terms of its viscosity: it may, however, be misleading to picture the mechanism as unagitated molecules creeping along against a viscous drag on their surface. By analogy we shall expect a relaxation time for dipole polarization when the applied field is removed. But here the mechanism is much more difficult to picture, since it does not consist of bulk transfer of molecules. Thus, consider Fig. 10, in which the uniform angular velocity of the molecule about the axis through P has a pulsation imposed on it by the action of the applied field. If the field is removed the pulsation disappears instantly, without a transient term, and uniform rotation continues at the angular velocity obtaining at the instant when the field was removed: this would be equally true if the rotational energy were small and the molecule oscillated without complete rotation. Thus in a gas it would seem as if the polarization effect disappeared instantly but that each molecule was left with an energy which depended both on its previous collision and on the time between that collision and the instantaneous removal of the field. Thus the velocity distribution would be not quite Maxwellian and a finite time would elapse before this state was reached: the relaxation time looks to be one of velocities rather than of polarization. However, it is possible that this is apparent and not real, and that a polar gas which is not in an electric field exhibits no net polarization effect only if the velocity distribution is Maxwellian. It seems likely that the redistribution of velocities would result in a slight increase of temperature of the whole, and thus we should encounter dielectric loss.

Once grant a time factor in the attainment of the Boltzmann distribution function and I think we are committed to a dipole polarization which dies away exponentially with time when the field is removed; and thus a condenser with a polar dielectric must be representable by some equivalent circuit such as that of Fig. 4(*b*), and in which the equivalent leakage resistance must be related in some manner with the macroscopic viscosity of the fluid dielectric. If this be so, then electrical tests alone cannot suffice to distinguish between energy loss due to Wagner conduction and due to polar molecules. In the first the heating is caused by ions drifting through liquid droplets and thereby adding to their thermal agitation, and in the second it is caused by a slight redistribution of velocities. It is important to realize that curves typified by that of Fig. 6 are likely to arise from almost any mechanism and hence do not alone suffice to disclose the mechanism: it is all the more strange that they are unfamiliar in practice.

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* The number 3 and not 2 has appeared previously through the process of integration over a sphere.

So far we have in the main considered a gas rather than a liquid. In a liquid the molecules are closely packed and always exert large forces on one another and thus are never free in the sense that they are free between collisions in a gas. Their thermal motion is vibratory with a frequency some 10^6 times as great as the collision frequency in a gas. An analysis such as that used in relation to Fig. 9 will now be very inadequate, for the twisting dipole is in this case acted on both by the strong electric field of its neighbours and by the applied field, and also only a tiny portion of a revolution will occur between consecutive collisions. The inter-molecular field will, on the average, be expressible in terms of kT, since there is equipartition of energy between the potential and the kinetic energy. The Boltzmann distribution theorem is still applicable for a dilute solution of dipoles in a non-polar liquid, and hence it may well be that now the equation $\overline{M} = \frac{\mu\epsilon}{3kT}$ is fulfilled because the dipole is restrained from turning by a true resisting couple as well as by its inertia, whose effect now may be relatively small. Debye[5] introduces a resisting couple due to viscosity, but dismisses this very difficult concept in very few words. The term "viscosity" is a way of expressing what happens during the brief intervals of collision, and it refers to an effect which is absent between collisions. It looks rather as if Debye were reckoning the same thing twice over, calling it first the disordering effect of thermal agitation and then the resisting effect of viscosity. No doubt the apparent confusion can be resolved and the treatment justified.

Debye's treatment leads me to the picture of a dipole devoid of rotational energy and whose rotation is resisted both by a restoring couple $2kT$ and by a viscous couple ζ; thus giving a time-constant or relaxation time $\tau = \zeta/(2kT)$. It is important to know whether this picture is intended to be real or merely an equivalent. Be this as it may, however, it follows from what has been said just previously that the curves of $\tan\delta$ and frequency will be the same as for the Wagner mechanism (see Fig. 5), and that $\tan\delta$ will be a maximum when $p\tau = 1$.

Debye wishes to make a rough estimate of τ and in order to do so applies Stokes's formula for the viscous drag on a sphere; as a first approximation he writes

$$\zeta = 8\pi\eta a^3 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where a is the radius of the polar molecule and η is the viscosity of the liquid. He is careful to explain that the treatment is rough and drastic, but reminds his readers that a similar treatment has already been found successful in calculations of the mobility of ions in an electrolyte.

Many measurements have been made of the energy loss in liquid dielectrics containing a dilute solution of polar molecules. All have fulfilled precisely the behaviour typified by Fig. 5: and in fact the curves of $\tan\delta$ and frequency of these dielectrics have had that form which theory would suggest but which is encountered so seldom in practice. Moreover, and somewhat surprisingly, equation (8) has been shown by Bridgeman[6] to be valid for large molecules such as zein, gliadin and haemoglobin.

But perhaps no measurements have illustrated the effect more strikingly than those made by W. Jackson on a sample of the commercial dielectric known as permitol (a mixture of isomers of tetrachlorodiphenyl) and reproduced in Fig. 11. Here $\tan\delta$ is plotted as a function of temperature at eight frequencies in the range 50 to $1 \cdot 09 \times 10^7$ c./sec. Heretofore we have regarded the temperature as constant and the frequency as variable, but brief consideration will show that these conditions differ little from those of constant frequency and variable temperature. We have seen that the governing factor is the ratio of the periodic time of the field to the time-constant (relaxation time) of the dielectric. The relaxation time depends on the viscosity, and this is markedly a function of temperature. Hence if the temperature is varied the relaxation time is variable over a wide range, and somewhere in this range it is likely to agree with the periodic time of the field: if the temperature is constant we must vary the frequency till it agrees with an assigned relaxation time. Here, in Fig. 11, we see at last a commercial dielectric behaving in the manner predicted by Wagner but almost unrealized heretofore. That the mechanism is not due to the conduction process of Wagner, but to the dipole rotation of Debye, is in a sense hardly relevant. Here is a dielectric which misbehaves only in a very small range of temperature, instead of behaving with the uniform mediocrity so familiar to engineers. Thus, suppose permitol is to be used at a frequency of 1 kc./sec.: it is only necessary to keep its temperature above about 20° C., and it is an admirable dielectric. If it is to be used at 1 Mc./sec. it must be kept above 50° C. or below 10° C. Though the material is capable of being very bad, it is only necessary to use it with knowledge and understanding to obtain a first-rate dielectric.

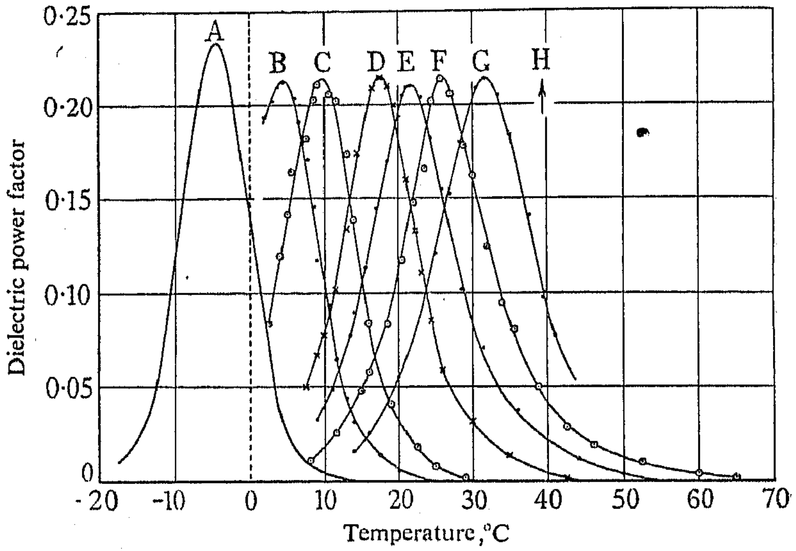

Fig. 11*

Frequency values, in cycles per sec.: A, 50; B, 10^3; C, 6×10^3; D, 10^5; E, $2 \cdot 95 \times 10^5$; F, $9 \cdot 5 \times 10^5$; G, $2 \cdot 75 \times 10^6$; H, $1 \cdot 09 \times 10^7$.

At each maximum in Fig. 11 the relaxation time of the dielectric is equal to $1/(2\pi f)$, and the relaxation time should be proportional to the viscosity. Hence if the reciprocal of the frequency is plotted against the temperature for the maximum value of tan δ, the resulting curve should represent, to some undetermined scale, the viscosity/temperature curve of the liquid; and this curve can be found by viscometer measurements. The full-line curve in Fig. 12 shows the measured viscosity of the permitol for temperatures greater than 10° C. (At temperatures less than about 10° C. permitol is a glassy substance and viscometry becomes impracticable.) The dotted curve in the same Figure shows log $1/f$ plotted against T, the scale being chosen so as to make the two curves agree at the point marked " a." This Figure shows conclusively that the relaxation time of the dielectric is proportional to the viscosity of the liquid, and that this is true even when the permitol has become a glassy solid. It is now clear that the behaviour of a polar liquid at every temperature and frequency can be predicted accurately if the curve of tan δ plotted against T is measured at any convenient frequency, and the viscosity curve is available. Recognition of this fact should be of great practical value to engineers. I feel it would obscure the issue to say that all this necessarily confirms Debye's treatment or his picture of dipole rotation, for I think the result is much more general than that. Granted only that the all-essential Maxwell–Boltzmann distribution needs time to establish itself, a polar dielectric must have a relaxation time: experiment shows that this time is proportional to the macroscopic viscosity. It would be somewhat surprising if it were otherwise, since the mechanism of viscosity, whatever it be, is one by which local agitations are distributed through the bulk medium, and we are familiar with the close connection between the viscosity of an electrolyte and its resistivity, and also between the viscosity of a gas and its thermal conductivity.

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* This Figure is due to W. Jackson: *Proceedings of the Royal Society*, A, 1935 **153**, p. 161.

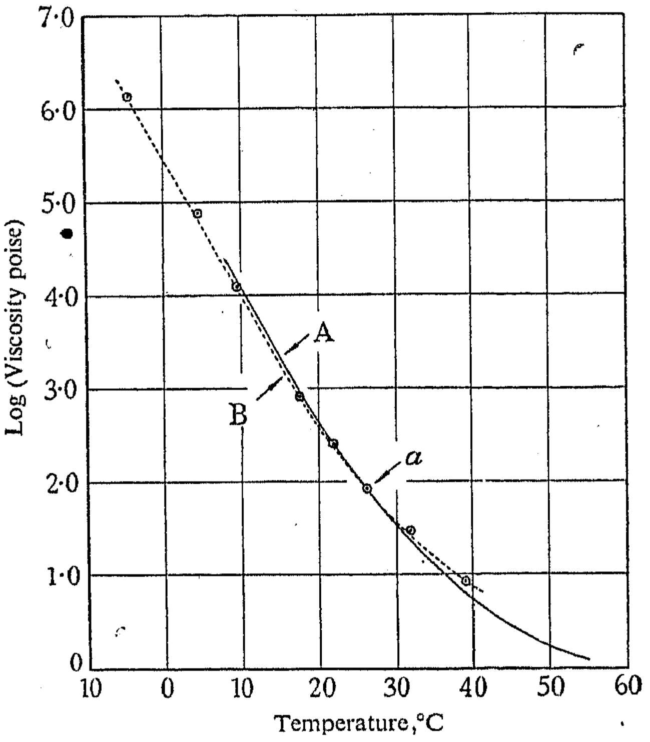


Fig. 12

A ——— measured viscosity/temperature curve.
B - - - ⊙ - - - calculated curve as determined from the position of the peaks of the curves in Fig. 2.

For the purpose of comparison the curves have been made to coincide at the point *a*.

Of course, we shall wish to go further and visualize, as Debye and (more recently) Andrade[7] have done, the mechanism of dipole rotation and the mechanism of viscosity respectively; but do not let us confuse the essential experimental facts, which are conspicuous by their simplicity, with these further reachings-out into a theory of the mechanism. We can use Fig. 12 to reach out like Debye and, by means of equation (8), to estimate the volume of the dipole molecule. Numerical computation from equation (8) yields a value of about 26 cubic Ångströms for this volume, whereas chemical information suggests the volume of the molecule must have been greater than 80 cu. Å. It is interesting to find that the two values are comparable: it is fruitless to speculate about the discrepancy until more is known about the inherent rigidity of the polar group in the molecule and until the mechanism of rotation has been established more firmly.

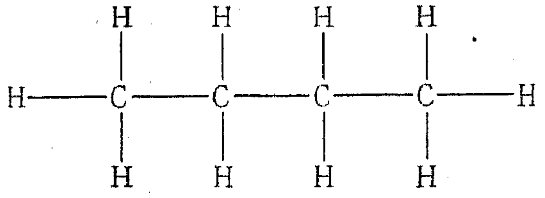
Granted the picture of a static and restrained dipole frozen into a sticky liquid, curves such as those of Fig. 11 are susceptible of vivid interpretation, as follows. When the liquid is very cold the dipoles are frozen tight and therefore cannot respond appreciably, and consequently there will be little energy loss. When the liquid is hot the dipoles can turn freely with very little friction and so once more there is very little loss: in intermediate stages the response will be finite and the friction considerable, resulting in considerable loss of energy. It is an instructive picture, but I suggest one which is better used as an aid to memory than as a picture of reality. It is a picture which would be very suitable if the dipole were a large colloid particle floating in a liquid: then the discrepancy between the mass of the particle and the surrounding molecules would make the Brownian rotations very small and leave the particle turning very slowly and acted on by a viscous drag calculable by Stokes's law. In such circumstances Debye's treatment would be fully justified and it is near these circumstances that Bridgeman, using haemoglobin in benzene, finds it fulfilled. But, when the dipoles are of the same size as the solvent molecules, surely we must be cautious lest we build too much on this picture.

(3) DIELECTRIC LOSS IN POLAR SOLIDS

The idea of viscous restraint on a polar molecule might lead one to suppose that rotation would be impossible in a crystalline solid, but such is not the case. Thus in 1924 J. Errera published a curve of tan δ against f for ice at − 40° C., and this showed a maximum and was generally reminiscent of the behaviour of a polar liquid. The behaviour of a polar solid is of great scientific interest and obviously has very great technical importance. We have been studying this problem in the Engineering Laboratory of the University of Oxford since 1933 and have been able to make some contributions to knowledge which may some day be helpful in the manufacture of dielectrics. Since there was scarcely any information available, it seemed essential to seek for it without any regard as to whether the materials examined had any great technical importance: in fact, to study the basic theory of dielectrics rather than to search blindly for materials which were good in the technical sense.

Thus the line of attack was to study the effect of adding known and chemically pure polar substances to a non-polar solid: the first problem was to find possible materials. Those chosen were esters added to paraffin wax. Since we hoped to deduce from electrical measurements the antics of certain molecules in the presence of others, it is essential to know what the molecules look like: hence a digression on molecular structure is necessary. The chemical formula of a paraffin wax is C_nH_{2n+2}, and the chemist often draws it like a picture of a centipede (Fig. 13). The formula and the picture imply that a range of substances can be made by the process of hooking together the appropriate number of identical links, as one might form a chain. Thus we might visualize pentane as a chain 5 links long and ordinary paraffin wax as a chain of about 26 links: that pentane is a liquid at room temperature and paraffin wax a very hard solid is irrelevant so far.

For further information we must go to X-ray crystallography, which in a very different scale of wavelength is a process very similar to our radio technique. Thus it would clearly be possible, though laborious, to deduce the spacing of an antenna array by directing at it a parallel beam of radio waves and observing the polar diagram of the reflected signals: it would be necessary to presume only that the spacing was regular. In the X-ray process each antenna of the vast array is a single molecule or single atom or ion, and in a crystal the spacing turns out to be regular. In some substances all similar ions have the same spacing in every direction, and then the molecule of the liquid state has lost its identity and the crystal consists of a perfectly regular lattice structure. Carbon can take up three different structures: (i) diamond, which consists of carbon atoms regularly spaced in every direction; (ii) graphite, which consists of layers of regularly-spaced carbon atoms and may be said to consist of layers of sheet molecules, or if you prefer, of flat sheets one atom thick. (iii) carbon can also exist in bundles of rod molecules, and then we call it a paraffin (comparison with a Franklin array is here irresistible).* However, the length of the rods in the paraffin crystal is finite and definite, and depends on the number of carbon atoms in the molecule of the vapour phase, and thus it may be said that the molecule preserves its identity from vapour through liquid to solid. Indeed, the centipede diagram of Fig. 13 turns out to be a true picture.



$$\mathrm{H}-\overset{\mathrm{H}}{\underset{\mathrm{H}}{\mathrm{C}}}-\overset{\mathrm{H}}{\underset{\mathrm{H}}{\mathrm{C}}}-\overset{\mathrm{H}}{\underset{\mathrm{H}}{\mathrm{C}}}-\overset{\mathrm{H}}{\underset{\mathrm{H}}{\mathrm{C}}}-\mathrm{H}$$

Fig. 13

The vapour of paraffin wax consists of widely separated rod molecules, one atom thick: when the wax is solid these are packed regularly side by side and may appropriately be likened to packets of candles. The molecule can be represented by Fig. 14 (which is much more than a mere diagram), i.e. by a series of spheres fixed together so that their centres are all in one plane. Each sphere represents a carbon atom, and the measured value of the diameter is $1 \cdot 54 \times 10^{-8}$ cm.: the measured value of α is 109° 30′, which is the angle of a regular tetrahedron. The two hydrogen atoms belonging to each carbon are too small to show up in the X-ray picture, but undoubtedly they are situated at the other two apices of the tetrahedron having two apices at the two contact points on each sphere. The centre distance between the axes of successive rods is either $4 \cdot 08$ Å or $3 \cdot 67$ Å, and the clearance between the ends is $2 \cdot 1$ Å. These distances are the same in every paraffin; it is only the chain length which changes from member to member of the series. When the crystal is heated the length does not expand, but the side spacings increase, one at three times the rate of the other. All the evidence shows that the rods are incredibly rigid—that they are properly called rods and not strings. It is proper to say they are as rigid as a diamond, for the work required to break the bond between two carbons is known to be the same as in a diamond and is about 80 kilogramme-calories.

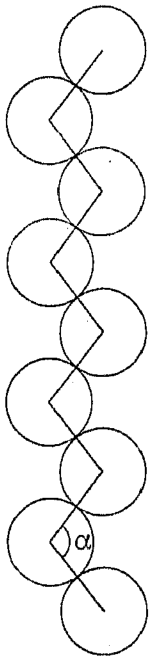
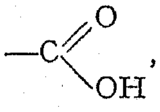
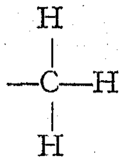


Fig. 14

The mechanical weakness of paraffin wax is due to the relatively small cohesion (Van der Waals forces) between the sides and ends of rods and not to the rods themselves, which are mechanically unbreakable, being thousands of times stronger than steel.

The symmetry of each atom precludes the possibility of a permanent dipole; and no paraffin, whatever its length, is in the least degree polar. The dielectric constant can be due only to strain of the valency electrons in each atom, and, barring density effects, it is obvious that all paraffins must have the same dielectric constant. Thus paraffin wax is essentially a non-polar crystalline solid having a precisely known and simple structure: it is admirably suited to our experiments if we can find polar materials to add to it.

If the last link in the chain consists of a carboxyl group  $-\mathrm{C}{\overset{\mathrm{O}}{\underset{\mathrm{OH}}{}}}$, vice a methyl group  $-\overset{\mathrm{H}}{\underset{\mathrm{H}}{\mathrm{C}}}-\mathrm{H}$, the rod mole-

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* It is amusing to think that if we could sever every horizontal tie rod in a diamond without severing vertical ties, the net result would be, roughly speaking, to convert the diamond into paraffin wax.

cule is called a fatty acid: the rod to which the carboxyl is attached may have any length from zero (in formic acid) to 17 carbons (in stearic acid, etc.). The carboxyl group is not symmetrical and, according to expectation, has a permanent dipole.

Now it seems probable that an *acid rod* would pack in without much fuss among paraffin rods, provided perhaps it was shorter than, but not too short compared with, the paraffin rods. The Van der Waals force from the polar end will be considerable and may cause some misfit: in the acid crystals it tilts the rod through about 40°, and this may perhaps be the angle between the direction of the dipole and the axis of the rod. The crystal is divided into unit cells each of which very often contains four molecules. Now it may be that, say, three acid and one paraffin fit into a unit cell of the wax better than two and two, or one and three, but whatever the arrangement of a unit cell which does contain an acid molecule it would seem probable that any degree of solid solution should be possible up to that amount in which every cell has its complement.*

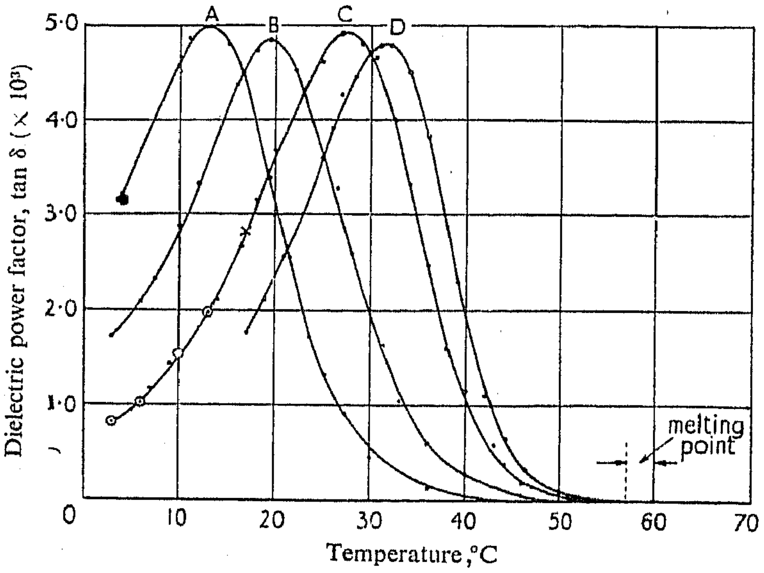

Fig. 15

Frequency of measurement, in cycles per sec.: A, $6 \cdot 6 \times 10^5$; B, $2 \cdot 8 \times 10^6$; C, $7 \cdot 78 \times 10^6$; D, $1 \cdot 42 \times 10^7$.

If the last link on the chain is hydroxyl, —OH, the rod molecule is called an alcohol: thus structurally the difference between acids, alcohols and paraffins is extremely small, and indeed, the higher members look indistinguishable in bulk. The unsymmetrical hydroxyl group is also polar, and so here we have another range of substances which should mix happily into the wax. But alcohol rods and acid rods will readily weld together and form a straight, rigid rod having a dipole at the junction of the two parts: such a rod is called an ester. Because the dipole in the ester, with its accompanying Van der Waals field of force, is not at the end of the rod, it seemed reasonable to suppose it might fit better than an alcohol or an acid into the paraffin lattice. For this reason and others it was decided to use esters as the polar material to mix into the wax.

A high-grade paraffin wax of setting point 57°–60° C. was available: this setting point indicates that the dominant chain-length was about 26 carbon atoms. The material spermaceti consists dominantly of the ester cetyl palmitate, which has the polar COO group with 15 carbons on one side and 16 on the other: thus it is almost symmetrical, and the total length of the rod is equivalent to about 33 carbon atoms, and this is about 25 % longer than the wax molecules. Spermaceti and wax appeared to mix without difficulty, and it was hoped that they went into solution.* It was found that the value of $\tan\delta$ for the wax was not greater than 5×10^{-5} in the temperature range 0°–80° C. but that the addition of $4 \cdot 5$ % of spermaceti increased this one-hundredfold at a sharp maximum at a temperature depending on the frequency. A set of $\tan\delta$/frequency curves is shown in Fig. 15, and they are characteristic of a polar liquid. A curve connecting $\tan\delta_{max}$ and the concentration is exhibited in Fig. 16, and this shows that the dielectric loss is certainly due to the addition of the permanently polarized ester molecules.[10] The magnitude of the dipole moment can be deduced from Fig. 15, and the figure obtained was $12 \cdot 5$ % less than that found from measurements in benzene solution. There can thus be no doubt that the behaviour of polar molecules in a crystal lattice is not substantially different from their behaviour in liquid solution. The relaxation time of a given molecule in a given wax at a given temperature is still a measurable quantity, but we can no longer follow Debye and estimate what this relaxation time will be because, firstly, there is so far no certain knowledge that the dipole cannot swivel round the long axis of the molecule without twisting with it the long carbon chains on either side. Secondly, with or without this uncertainty, is it fair to treat the molecule either as a sphere or as a smooth rod? Lastly, and perhaps most pertinently, what magnitude is to be assigned to the viscosity in the solid state? Viscosity, according to definition, no longer has any meaning, yet presumably there will exist a dissipative mechanism in the solid which plays a role corresponding to that of viscosity in a liquid. If in Fig. 15 the temperature at the maxima is plotted against the logarithm of the reciprocal of the corresponding frequency, the points lie on a straight line. This implies that the parameter which for convenience we will still call viscosity varies exponentially as the reciprocal of the

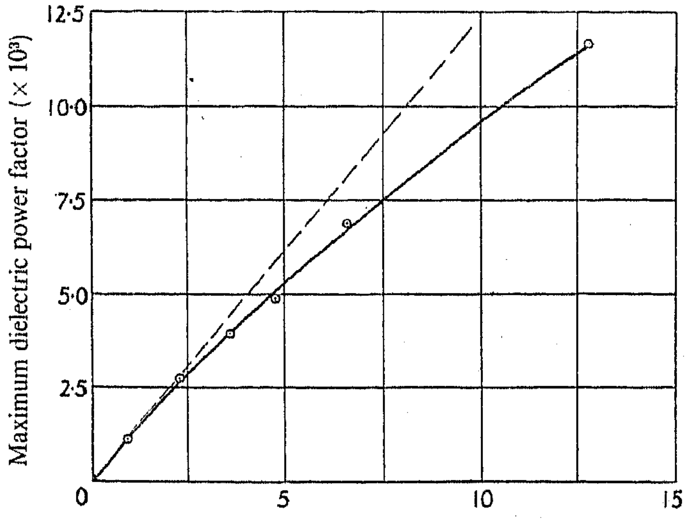

Fig. 16

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* In point of fact, however, the evidence of X-ray and melting-point phenomena shows that the active ends of the acid rods unite and form a double molecule: possibly acids will form solid solutions with paraffins only if the acids are less than half as long as the paraffin.

* The chemical side of these first experiments was guided by Dr. F. C. Frank, and the electrical work was performed by Prof. W. Jackson.

temperature: in this respect there is continuity with the liquid state. If the volume of the whole molecule is substituted in Debye's equation, equation (8), then the viscosity in the solid wax at 40° C. is less than in the molten wax at 80° C.: if the volume of the rotating part is taken to be that of the polar group alone the apparent viscosity is about equivalent to that of castor oil. With liquid permitol the measured viscosity was a useful guide for predicting the behaviour of the dielectric, but for the solid there is no such guide available at present. At least it is possible to investigate the effect of the size of the molecule by using esters of different length: if the polar group is free to swivel in the rod, the length of the rod should be of no consequence. The result of such an investigation[11] is summarized in Fig. 17, and relates to esters ranging from 21 to 33 carbon atoms long. At any given temperature, say − 20° C., the viscosity of the wax will have the same value for the rotation of all the esters. Yet for butyl palmitate (21 carbons), $2\pi\tau$ is equal to 0·2 microsec., and for cetyl palmitate (33 carbons) $2\pi\tau$ is equal to 98 microsec. Thus the length of the rod does affect the relaxation time, and hence the whole rod must turn rigidly with the dipole. These measurements have thus disclosed something about the molecular structure of ester molecules, namely that the whole rod is rigid round its long axis; at every bond both C—C and C—O.

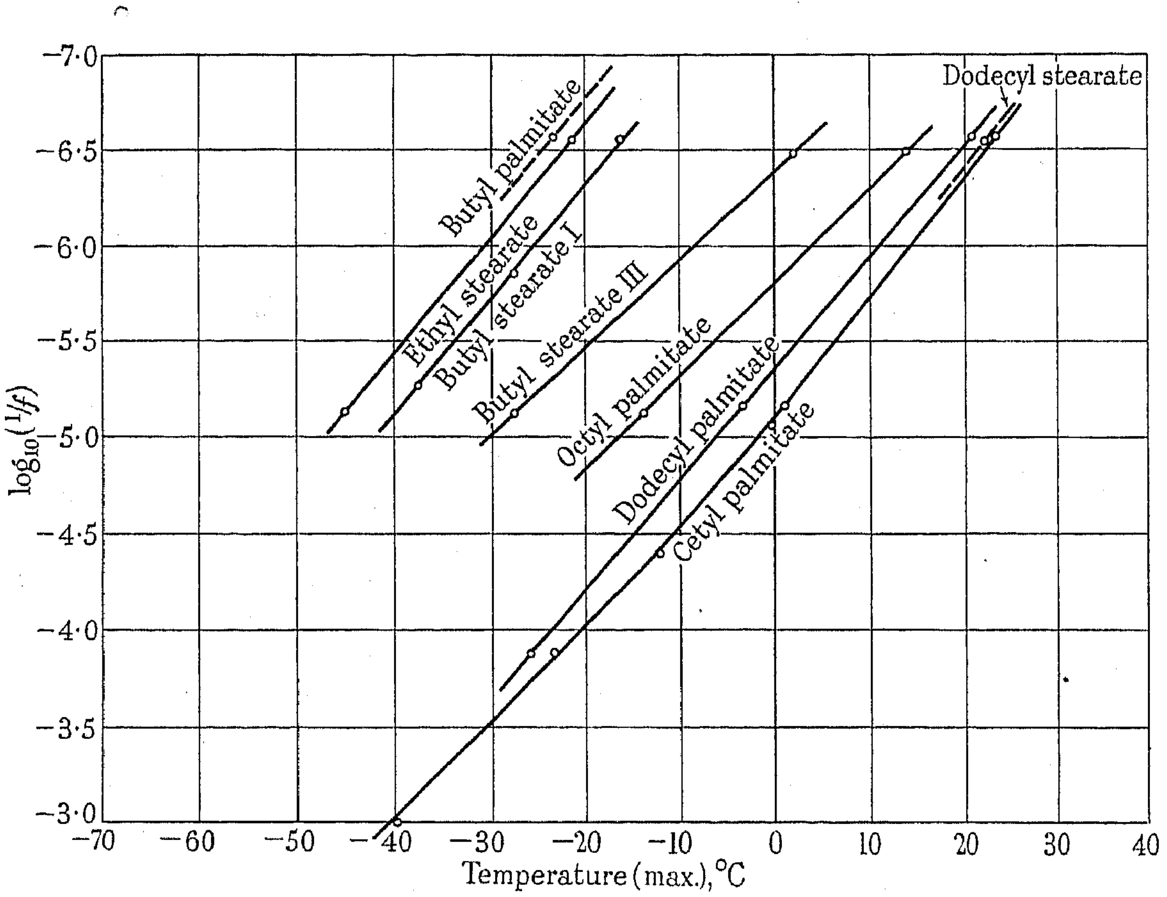

Fig. 17

We have described the formation of an ester as the welding-together of an alcohol and an acid chain. Thus an ester of given total length can be made by welding an alcohol to an acid of equal length, or a short alcohol to a longer acid: in other words, the weld (that is, the dipole) can be placed at any desired point along the length of the finished rod. Thus it is possible to see whether the relaxation time depends on the position of the dipole: the answer is that it does not depend on the position but only on the total length. Thus ethyl stearate and butyl palmitate are both 21 carbons long; in the first the dipole is placed at 10 % of the length, and in the second at 20 %. Yet the electrical behaviour of these two esters has been found to be indistinguishable (see Fig. 17), and this is not the only test of this property which has been made. But since increasing the chain length from 21 to 33 carbons increases $2\pi\tau$ by 500 times, it is quite certain that the relaxation time cannot be proportional to the volume of the rod. Hence we now know that the very form of Debye's equation for τ is invalid in the solid state.

There is one more very fascinating experiment to recount, and one which has obvious possibilities of technical use as well as being of deep scientific interest. Seeing we have proved that the dipole is rigid with the rod, it should be possible to obtain either twice the effect or zero effect by making a rod having two dipoles facing either in the same or in opposite senses. The sense can be arranged by using an odd or an even number of carbon atoms between the two dipoles: this is illustrated by Figs. 18(*a*) and 18(*b*) for dioctyl sebacate and dioctyl azelate respectively. Experiment showed[8] that the azelate gave the familiar sharp maximum of tan δ (0·33 % for 3·73 % concentration), whereas the sebacate gave no maximum and tan δ was less than 0·025 % between − 25° and + 60° C. Hence the oppositely directed dipoles were able to neutralize one another and render the material effectively non-polar so far as dielectric loss is concerned. Such a disposition of dipoles has obvious technical possibilities for the production of low-loss materials: these have been explored tentatively for certain alkyd resins.[9]

(4) SOME PROBLEMS YET TO BE SOLVED

This Address has reached already an inordinate length, yet there is still much fundamental work that has not been described. I must hope later to contribute to the *Journal* a paper on electrical and mechanical properties of paraffin waxes both with and without polar molecules. With this Address as an introduction I could then discuss the mechanism of the phenomena which the electrical engineer must try to visualize if he hopes to make low-loss materials having assigned qualities. It is fairly certain he will have to use polar materials, because so very many organic substances are naturally polar. Also it seems to me that he will want to avail himself of the powerful Van der Waals forces of the dipoles in order to raise the melting point and increase the mechanical strength. He dislikes their electrical properties, and therefore he will render them innocuous as regards dielectric loss by using them in opposed pairs; or he will take steps to adjust their relaxation time so that maxima will be situated at a temperature far below, or far above, the working range. May one put it that he will gladly use rivets in his structure but either he will close them tightly or lubricate them freely to avoid friction loss. But before he can do that he must think out the mechanism of viscosity in a solid and learn the effect on the relaxation time of the shape of the molecule.

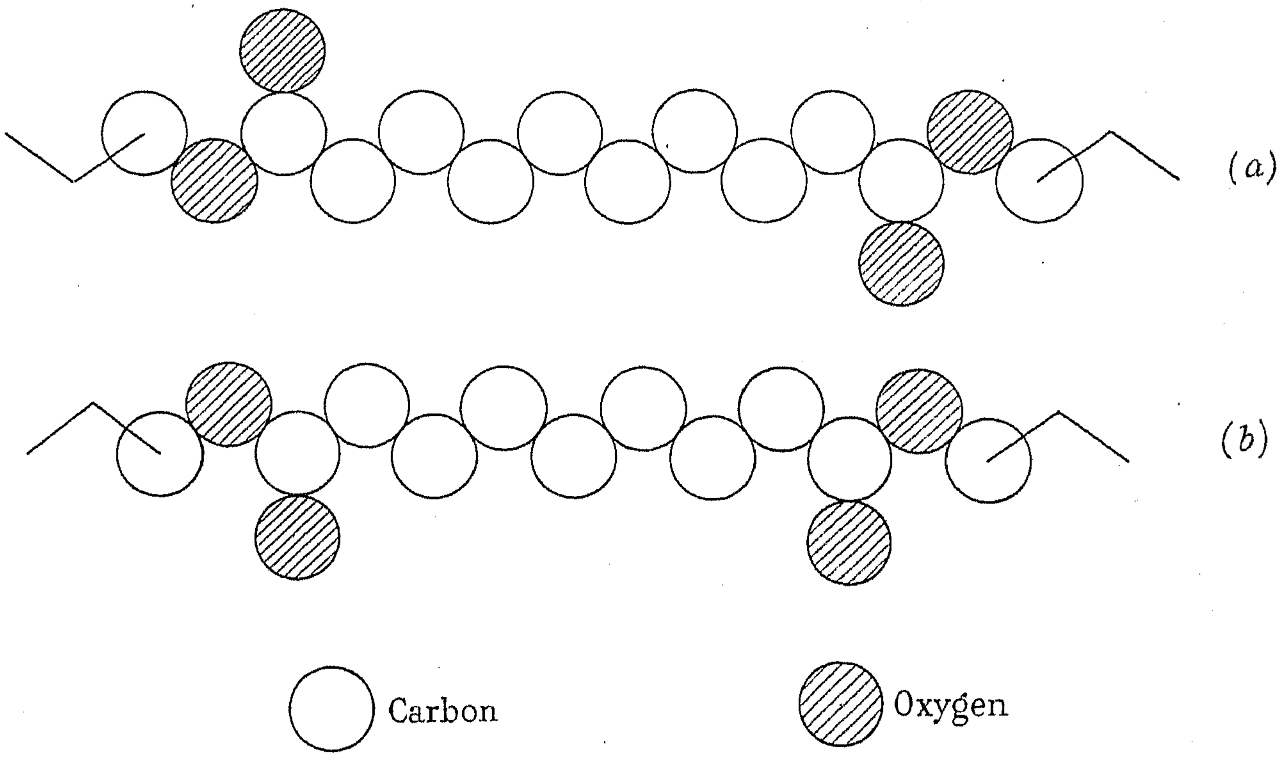

Fig. 18.—Part of the molecules of (*a*) dioctyl sebacate and (*b*) dioctyl azelate, showing how the dipole moments are in the opposite sense in (*a*) and in the same sense in (*b*).

I will close on an imaginative note. May the day come when the mechanical, electrical and thermal properties of a desired dielectric are first specified; then the drawing office is instructed to design a molecule of the size and shape which will do the job; then this working drawing is handed over to the *chemical smith* to hammer up sufficient of the desired parts; and finally the erectors put it all together. The picture is no doubt fantastic, but not impossible. If it is fulfilled, we shall work with more mental satisfaction and certainty than in the days of the witch's cauldron of metaphorical herbs and frogs' legs which reigned supreme till a few years ago.

LIST OF SYMBOLS

a = radius of orbit.
C_1, C_2 = capacitances.
d = distance between condenser plates.
e = electronic charge.
f = frequency.
g = acceleration due to gravity.
$\mathbf{I}$ = moment of inertia.
$K = C_1C_2/(C_1 + C_2)$.
k = Boltzmann's constant.
l = length.
M = electric moment of atom due to field.
m = mass.
N = number of molecules.
n = number of spheres per unit volume.
P = pressure.
$p = 2\pi f$.
R = resistance.
$s \equiv mgl/(\frac{1}{2}ml^2\omega^2)$.
T = absolute temperature.
t = time.
v = fractional volume of moisture in dielectric.
x = distance moved by proton.
$y = \alpha\epsilon/(2kT)$.
$z = \dfrac{\text{molecular volume from measurements of } \kappa}{\text{molecular volume from non-electrical measurements}}$
α = spring constant.
$\delta = \pi/2 -$ (lead angle of combination shown in Fig. 5*b*).
ϵ = electric field.
θ = angular displacement.
κ = dielectric constant.

LIST OF SYMBOLS—*continued.*

μ = inherent electric moment of molecule.
ρ = density of material, and electrical resistivity.
ω = angular velocity.

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THE ELECTRIC AND MAGNETIC FIELDS OF A LINEAR RADIATOR CARRYING A PROGRESSIVE WAVE*

By F. M. COLEBROOK, B.Sc.

(*Paper first received* 13*th January, and in revised form* 29*th May*, 1939.)

SUMMARY

The vector- and scalar-potential method of calculating electric and magnetic fields is applied to a straight conductor carrying a progressive wave of current. It is shown that unless the conductor is assumed to be terminated by charges which satisfy the condition of electrical continuity at the ends, the calculated field has an anomalous and impossible character. Such charges need not be included in the case of closed circuits of conductors carrying progressive waves. The mutual cancellation in such cases of the anomalous features associated with each linear element is confirmed by analysis of a particular case. It is pointed out that a linear progressive wave radiator has only one axis of symmetry, i.e. the radiator itself, differing in this respect from a standing-wave radiator, which is symmetrical about the equatorial plane. The consequent asymmetry of the field from a progressive wave radiator is confirmed by analysis.

The field system associated with a progressive wave of current is regarded as a more fundamental conception than that due to a standing wave, since any standing-wave system of currents is resolvable into positive and negative travelling waves. This feature is illustrated by the synthesis of the travelling wave fields into the known form for a dipole standing-wave radiator.

The radiation resistance of a progressive-wave radiator is calculated by Pistolkors's method, and is shown to be equal to that for a standing-wave radiator when the length of radiator is an integral number of half wavelengths.

(1) INTRODUCTION

The author has recently had occasion to consider the design of a rhombic transmitting antenna for a particular purpose, and was thus led to study in some detail the nature of the field from a linear conductor carrying a progressive wave of current. In doing so he noted certain essential points of difference between conductors carrying progressive waves and conductors carrying standing waves. These have, no doubt, already been appreciated by the writers of the various papers on rhombic antennae which are now available (see Bibliography), but the present author has not found them explicitly dealt with in any of these papers. In view of the considerable and growing importance of the subject, it has been thought desirable to emphasize some of these difficulties and distinctive features in the present paper, which is analytical in character and is intended as a guide in future design and experimental work.

(2) ANALYTICAL BASIS

For the purpose of this paper, the method of analysis used by Pistolkors in his well-known paper on radiation resistance,† i.e. the derivation of the field from scalar and vector potentials, has some advantage over the more compact and elegant formulation given by Bechmann‡ in terms of the Hertzian vector.

Pistolkors starts with the formulae

$$\mathbf{E} = -\operatorname{grad}\psi - \frac{1}{c}\frac{d\mathbf{A}}{dt} \quad . \quad . \quad . \quad (1)$$

$$\mathbf{H} = \operatorname{curl}\mathbf{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

E and **H** being the electric and magnetic fields at a given point in space due to a system of charges and currents which give retarded scalar and vector potentials ψ and **A** at the point. The particular system to be studied in the first instance is that illustrated in Fig. 1, which represents a straight conductor of length $2l$ and of negligible cross-section, situated in free space. The conductor carries a current which is a function of time and of the distance along the conductor, its value being i at the instant t at the point distant z from the origin (the centre of the conductor).

The retarded vector potential **A** at the point P due to the current in the whole length of the conductor is given in absolute units by

$$\mathbf{A} = \mu c\int_{-l}^{l}\frac{[\mathbf{i}]}{r}dz \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where μ is the permeability of free space, c the velocity of light, and **i** a vector of magnitude i and direction parallel to the conductor. The vector **A** is therefore parallel to the conductor. The square brackets signify the usual retardation, i.e. in calculating **A** for the instant t the value of i to be used is that for the instant $t - r/c$.

There is a distribution of linear charge density σ along the conductor given by

$$\frac{\partial\sigma}{\partial t} = -\frac{\partial i}{\partial z} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the retarded scalar potential at P due to this charge distribution is given by

$$\psi_0 = \frac{1}{\kappa}\int_{-l}^{l}\frac{[\sigma]}{r}dz \quad . \quad . \quad . \quad . \quad . \quad (5)$$

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* Official communication from the National Physical Laboratory.
Reprinted from *Journal I.E.E.*, 1940, **86**, p. 169.

† See Bibliography, (8).

‡ *Ibid.*, (3).

where κ is the permittivity of free space, i.e.

$$c^2 = \frac{1}{\kappa\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Initial calculations for a linear radiator carrying a progressive wave, based on the above values of $\mathbf{A}$ and ψ_0, lead to certain anomalous results; in particular, a radial component of electric intensity even at great distances from the radiator. Applied to systems in which such linear radiators were formed into closed circuits, however, the calculations lead to apparently quite correct results. It was suggested to the author by a colleague (Mr. W. Ross) that the explanation probably lay in the incomplete formulation of the scalar potential. This author is also indebted to Mr. I. H. Cole of the Bawdsey Research Station for calling his attention to a paper by J. Groszkopf* in which this point is very explicitly dealt with.

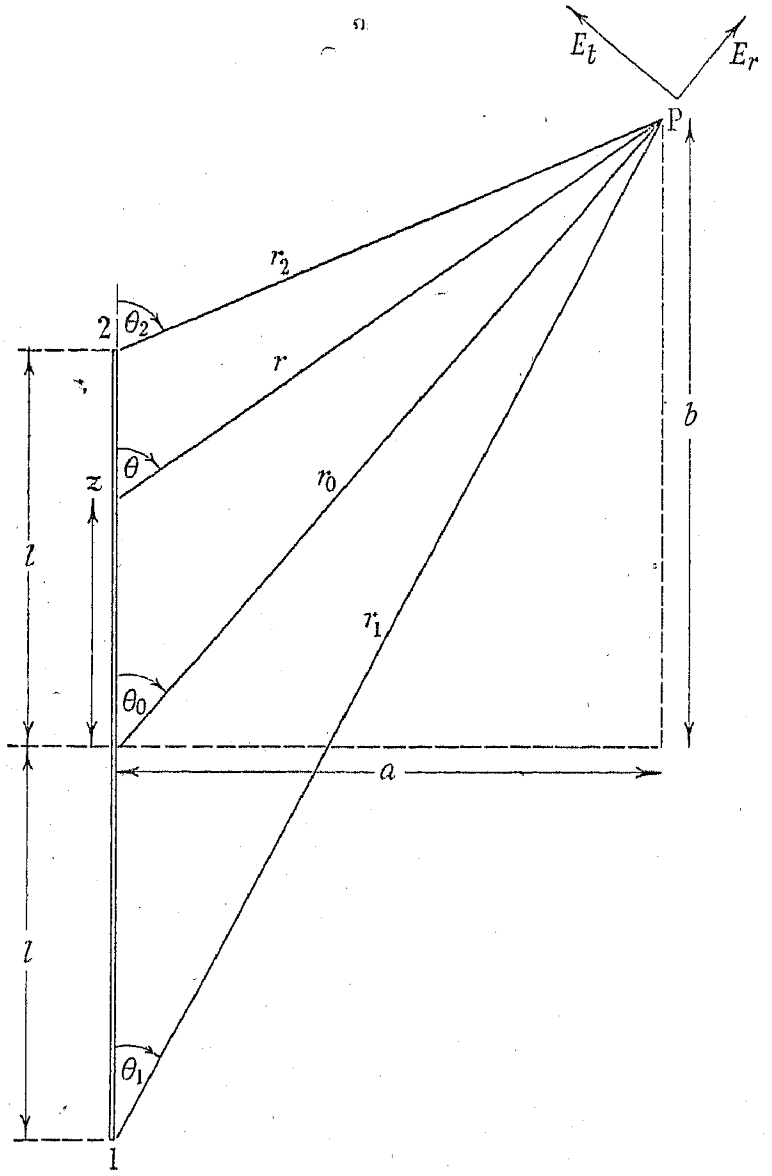

Fig. 1

Pistolkors was concerned with standing-wave radiators, having therefore zero current at the open ends. For such systems equation (5) completely represents the retarded scalar potential. In a linear radiator carrying a progressive wave, however, the amplitude of the current will be substantially uniform along its length. A single linear radiator carrying such a current, as shown in Fig. 1, is not therefore a physically self-consistent conception, and it is not surprising that anomalous results were obtained. A closed circuit of such conductors is, on the other hand, a physically self-consistent system. As Groszkopf points out, the single radiator can equally be made self-consistent by assuming it to be terminated by charges q_1 and q_2 at the ends 1 and 2, given by

$$\left.\begin{aligned} q_1 &= -\int i_1 dt \\ q_2 &= \int i_2 dt \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

i_1 and i_2 being the values of the current at the ends 1 and 2. These charges will give rise to retarded potentials ψ_1 and ψ_2 given by

$$\psi = \frac{1}{\kappa}\frac{[q]}{r}; \text{ suffixes } 1, 2 \quad . \quad . \quad . \quad (8)$$

and the total scalar potential is therefore

$$\psi = \psi_0 + \psi_1 + \psi_2 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

as defined by (5) and (8).

The significance of these additional terms in the determination of the field will be shown later.

Thus far, the only simplifying approximation made is that the diameter of the wire is negligible compared with all the other lengths involved.

It will now be assumed that the current distribution takes the form of an undamped progressive wave which, for simplicity, will be expressed in the exponential form

$$i = I_0 e^{j(\omega t + \alpha)} e^{-jmz} \quad . \quad . \quad . \quad . \quad (10)$$

where $m = 2\pi/\lambda_c$, λ_c being the wavelength of the current wave along the conductor. In practice, however, the conductor will have a certain ohmic resistance, with a consequent small attenuation of the current wave. An even larger factor is the attenuation due to the radiation from the conductor, which, as will be shown later, exists for a progressive wave just as it does for a standing wave. (The statement to the contrary by Everett and Byrne in 1929* was made prior to the introduction and use of progressive-wave transmitting-aerial structures and would presumably no longer be maintained by them. There appears to be no valid theoretical justification for it.) Thus, for an exact representation of a physically realizable system, it would be necessary to assume a current distribution of the form

$$i = I_0 e^{j(\omega t + \alpha)} e^{-(n + jm)z} \quad . \quad . \quad . \quad (11)$$

The resulting integrals cannot, however, be expressed in terms of known and tabulated functions. As a first approximation, therefore, it is necessary to neglect the effect of attenuation, reserving it for later consideration. It is likely to have a relatively small effect on amplitude rather than on the general character of the field.

One further point should be noted at this stage. The assumed current distribution

$$i = Ie^{-jmz}$$

where

$$I = I_0 e^{j(\omega t + \alpha}$$

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* See Bibliography, (13).

* See Bibliography, (10).

represents a progressive wave of current travelling up the conductor from 1 to 2, i.e. in the positive direction of z. Such a system has only one axis of symmetry, namely the conductor itself. A similar conductor carrying a standing wave can, as is well known, be regarded as carrying equal and opposite progressive waves, and its field will therefore be symmetrical with respect to the equatorial plane in direction and amplitude. No such symmetry can be expected for the progressive-wave radiator and does not, in fact, exist. The point is sufficiently obvious when stated in this way, but the author fell into error in one instance through overlooking it. Further, in all combinations of linear conductors carrying progressive waves it is the direction of the wave, rather than that of the current, which matters. Consider, for example, the rectangular structure shown in Figs. 2(a) and 2(b), in which progressive waves are obtained by a suitable choice of the terminating impedance. A consistent sign convention for the direction of the current would be as shown by the arrows in Fig. 2(a). The wave directions, however, are as shown in Fig. 2(b), and it is found that the field formulae depend on the relation between the direction of the current and the direction of the travel of the current wave.

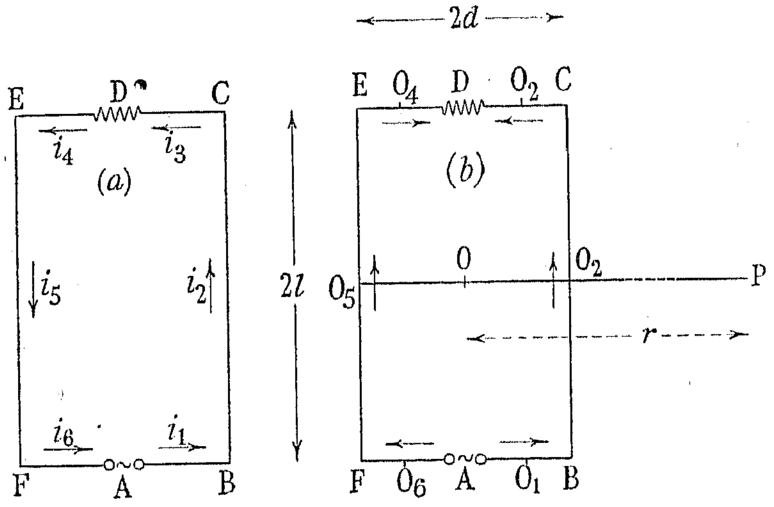

Fig. 2

For the assumed current distribution, it is easy to show that

$$[\mathrm{i}] = \mathbf{I}e^{-j\left(\frac{\omega r}{c}+mz\right)} \quad . \quad . \quad . \quad . \quad (12)$$

where $\mathbf{I}$ is a vector of magnitude $I = I_0e^{j(\omega t+\alpha}$ and direction z. Also

$$[\sigma] = \frac{m}{\omega}Ie^{-j\left(\frac{\omega r}{c}+mz\right)} \quad . \quad . \quad . \quad (13)$$

It is now necessary to make the further simplifying assumption that the wavelength λ_c along the conductor is the same as that in free space, $\lambda = 2\pi c/\omega$. This assumption is known to be very approximately correct and is, moreover, consistent with the assumed absence of attenuation. It is desirable, however, that all such assumptions should be clearly stated and appreciated. This gives

$$m = \frac{2\pi}{\lambda} = \frac{\omega}{c} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and the expressions for the current and charge density simplify to

$$[\mathrm{i}] = \mathbf{I}e^{-jm(r+z)} \quad . \quad . \quad . \quad . \quad (15)$$

and

$$[\sigma] = \frac{1}{c}Ie^{-jm(r+z)} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Further (see Appendix 2)

$$[q_1] = -\int[i_1]dt$$

$$= -\frac{1}{j\omega}Ie^{-jm(r_1-l)} \quad . \quad . \quad . \quad (17)$$

and

$$[q_2] = \int[i_2]dt$$

$$= \frac{1}{j\omega}Ie^{-jm(r_2+l)} \quad . \quad . \quad . \quad . \quad (18)$$

For the above values of i and σ, it is easily shown that in practical units,

$$\psi_0 = 30I\,.\,F \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and

$$\frac{1}{c}\frac{d\mathbf{A}}{dt} = jm30I\,.\,F \quad . \quad . \quad . \quad . \quad (20)$$

where

$$F = \int_{-l}^{l}\frac{e^{-jm(r+z)}}{r}dz \quad . \quad . \quad . \quad . \quad (21)$$

The detail of this integration is contained in one of the papers by Bechmann, and is outlined in Appendix I for convenience of reference.

(3) THE CO-ORDINATE SYSTEM

The most convenient co-ordinate system is spherical-polar, with the origin at the centre of the radiator, the co-ordinates r_0 and θ_0 of P being as shown in Fig. 1. However, the r and θ shown for the point z in Fig. 1 are not required after integration and it will therefore be permissible and convenient to drop the zero suffixes in r_0 and θ_0 in the formulae for the fields at P. On account of axial symmetry the system is effectively two-dimensional and the ϕ co-ordinate of P is not required.

(4) THE ELECTRIC AND MAGNETIC FIELDS

It will be convenient to distinguish between the component of the total electric field at the point P which is due to the distributed current and charges of the linear radiator, and the component which arises from the fictitious terminating charges introduced in order to make the system physically consistent, for, as will be shown later, only the former need be taken into account in dealing with closed-circuit structures of linear radiators. For convenience, the terminal-charge component will be distinguished by a suffix, q, i.e. the two main components will be written E and E_q.

For distant points, the most convenient resolution of the electric field is into radial and tangential components E_r and E_t; $(E_q)_r$ and $(E_q)_t$, as shown in Fig. 1. For some purposes, e.g. determination of radiation resistance by Pistolkors's method, a more convenient resolution is into components parallel and perpendicular to the radiator, i.e. E_z and E_x, etc.

The vector **H** is everywhere perpendicular to the plane defined by r and the radiator, and will otherwise be specified by its magnitude H.

An outline of the evaluation of these various components is given in Appendices 2 and 3. The results are:—

$$E_r = \frac{-30I}{r}\left\{\frac{r_2 - l}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l}{r_1}e^{-jm(r_1-l)}\right\} \quad (22)$$

$$E_t = \frac{30I}{r}\cot\theta\left\{\frac{r_2 - l + r/\cos\theta}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l + r/\cos\theta}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (23)$$

$$(E_q)_r = 30I\left\{\left(1 - \frac{j}{mr_2}\right)\left(\frac{r - l\cos\theta}{r_2}\right)\frac{e^{-jm(r_2+l)}}{r_2} - \left(1 - \frac{j}{mr_1}\right)\left(\frac{r + l\cos\theta}{r_1}\right)\frac{e^{-jm(r_1-l)}}{r_1}\right\} \quad . \quad (24)$$

$$(E_q)_t = -30Il\sin\theta\left\{\frac{e^{-jm(r_2+l)}}{r_2^2} - \frac{e^{-jm(r_1-l)}}{r_1^2}\right\} \quad . \quad (25)$$

for the radial and tangential components of E, while for the parallel and perpendicular components

$$E_x = \frac{-30I}{r\sin\theta}\left\{\frac{r_2 + r\cos\theta - l}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + r\cos\theta + l}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (26)$$

$$E_z = 30I\left\{\frac{e^{-jm(r_2+l)}}{r_2} - \frac{e^{-jm(r_1-l)}}{r_1}\right\} \quad . \quad . \quad . \quad . \quad (27)$$

$$(E_q)_x = 30I \, . \, r\sin\theta\left\{\left(1 - \frac{j}{mr_2}\right)\frac{e^{-jm(r_2+l)}}{r_2^2} - \left(1 - \frac{j}{mr_1}\right)\frac{e^{-jm(r_1-l)}}{r_1^2}\right\} \quad . \quad (28)$$

$$(E_q)_z = 30I\left\{(r\cos - l)\left(1 - \frac{j}{mr_2}\right)\frac{e^{-jm(r_2+l)}}{r_2^2} - (r\cos\theta + l)\left(1 - \frac{j}{mr_1}\right)\frac{e^{-jm(r_1-l)}}{r_1^2}\right\}. \quad (29)$$

Finally,

$$H = -E_x \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

These formulae suffice for the complete determination of the field at any point in space due to any closed-circuit configuration of linear conductors carrying progressive waves, and also for the field due to any single conductor carrying standing waves, since the latter can always be resolved into its progressive-wave components. In this sense they are more fundamental than the usual formulae for standing-wave radiators.

(5) THE FORMULAE FOR DISTANT NON-AXIAL POINTS

In nearly all cases of practical interest, the point P is so distant that l is very small compared with r, and certain simplifications can be made in the above formulae. They must, however, be introduced with some care. It is, for instance, permissible to put

$$\frac{1}{r_1} = \frac{1}{r_2} = \frac{1}{r}$$

since this only involves the neglect of terms proportional to $1/r^2$ compared with terms proportional to $1/r$, but for the exponential indices a closer approximation must be used, i.e.

$$r_2 + l \simeq r + l(1 - \cos\theta)$$

and

$$r_1 - l \simeq r - l(1 - \cos\theta)$$

Then, neglecting terms in $1/r^2$ (i.e. the induction field) compared with those in $1/r$ (i.e. the radiation field)

$$E_r = \sin\{ml(1 - \cos\theta)\}\frac{e^{-jmr}}{r}60jI \quad . \quad . \quad (31)$$

$$E_t = -E_r \, . \cot\theta/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$(E_q)_t = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$(E_q)_r = -E_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

for radial and tangential components, and

$$E_z = -E_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$E_x = E_r\cot\theta/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$(E_q)_z = -E_r\cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

$$(E_q)_x = -E_r\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

for parallel and perpendicular components. Also

$$H = -E_r\cot\theta/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

$$= E_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

The formulae are not valid for axial points, for which separate consideration is required (see Section 7).

(6) THE SIGNIFICANCE OF THE TERMINAL CHARGES

The common factor e^{-jmr}/r, where $m = 2\pi/\lambda$, shows that the field at distant points is inversely proportional to the distance, a well-known characteristic of the radiation field from a point source. Further, the factor e^{-jmr}, which determines phase, is such that the direction of maximum rate of change of phase is radial, i.e. the wave is spherical at distant points and the direction of propagation is radial. It will be seen, however, that if the terminal charges had been omitted the wave would apparently have had a component of electric intensity in the direction of propagation, and, moreover, the magnetic field, instead of being equal to the electric field, would have been equal to the tangential component. The formulae show that this anomalous radial component is due to the omission of the terminal charges. In fact, when the terminal charges are included, they contribute, obviously, nothing to the magnetic field, and to the electric field a radial component which exactly cancels the anomalous component, the resultant total field having all the characteristic features of a regular spherical wave in free space.

If the formulae are applied to a closed structure, for example that shown in Figs. 2(*a*) and 2(*b*), the terminal charges are not required to make the system physically consistent, and there will therefore be a radial component of field for each linear radiator. On the other hand, the essential character of the system will not be changed if the circuit be opened at the ends of each linear element and completed electrically by assuming the appropriate equal and opposite charges to exist on the opposite sides of each gap. The calculated fields at distant points from each such terminated linear radiator will be free from the anomalous radial component. It follows from this that in calculating the distant field for a closed circuit carrying a progressive wave, the resultant of all the radial components at distant points will necessarily be zero, and there is in fact no need to calculate them, except as a means of checking the accuracy of the calculations. This is illustrated in Section (11) for the structure illustrated in Figs. 2(*a*) and 2(*b*).

(7) THE FIELD AT AXIAL POINTS

As stated in Section (6), it is not necessary to calculate the radial components of the electric field at distant points due to any system of progressive-wave radiators forming a closed circuit, except as a means of checking the formulae employed. In making any such calculations, however, it is necessary to note that a linear progressive-wave radiator differs from a standing-wave radiator with respect to the field at axial points.

Referring to (22) and (23) we have, for axial points above the radiator (i.e. $\theta = 0$),

$$r = r_2 + l = r_1 - l \quad . \quad . \quad . \quad . \quad (41)$$

and, on making these substitutions,

$$E_r = \frac{l}{r^2 - l^2} e^{-jmr} 60I \quad . \quad . \quad . \quad . \quad (42)$$

Thus the radial component is very small and vanishes as $1/r^2$. It is obvious on grounds of axial symmetry that the tangential component E_t must be zero. The result of direct substitution is actually indeterminate, but is easily shown to be zero by the usual methods.

For axial points below the radiator, however, i.e. for $\theta = \pi$, a different result is obtained. We have in this case

$$r = r_2 - l = r_1 + l \quad . \quad . \quad . \quad . \quad (43)$$

The tangential component E_t is zero as in the previous case, but the radial component becomes

$$E_r = -30Ie^{-jmr}\left\{\frac{e^{-j2ml}}{r+l} - \frac{e^{j2ml}}{r-l}\right\} \quad . \quad . \quad (44)$$

and if l is small compared with r this becomes

$$E_r = \sin 2ml \cdot \frac{e^{-jmr}}{r} \cdot 60jI \quad . \quad . \quad . \quad (45)$$

Thus there is an appreciable radial component of order $1/r$ from the back end of a linear progressive-wave radiator, a notable point of difference from the standing-wave radiator and one liable to be overlooked. An example of this is given in Section (12), where the field from a rectangular progressive-wave aerial structure is calculated. It is there shown that the field at P (Fig. 2*b*) due to the vertical members BC and FE contains a radial component of order $1/r$. The parts AF and CD of the horizontal members are, however, in a back-end-on position with respect to the point concerned, and it is found that the radial components from these opposite halves of the top and bottom exactly cancel the radial component due to the vertical members, giving a regular resultant field with no radial component.

(8) THE ASYMMETRY OF THE FIELD FROM PROGRESSIVE-WAVE RADIATORS

For distant points r, θ and r, $\pi - \theta$, which are symmetrical about the equatorial plane of a linear progressive-wave radiator, the tangential components of the electric field are respectively, from (31) and (32),

$$E_t(r, \theta) = -\sin\left\{ml(1 - \cos\theta)\right\}\cot\theta/2\frac{e^{-jmr}}{r}60jI \quad . \quad (46)$$

and

$$E_t(r, \pi - \theta) = -\sin\left\{ml(1 + \cos\theta)\right\}\tan\theta/2\frac{e^{-jmr}}{r}60jI \quad (47)$$

Thus the field is not symmetrical about the equatorial plane. A progressive-wave radiator has in fact only one axis of symmetry, i.e. the radiator itself. The difference already noted with respect to the axial points in the forward and backward directions is the extreme case of this asymmetry.

(9) FORMULAE FOR A REVERSED OR DOWNWARD WAVE OR CURRENT

The formulae in Section (4) refer to the system shown in Fig. 1 and to an upward wave of current, i.e. in the direction 1 — 2 represented by

$$i = I_0 e^{j(\omega t + \alpha)} e^{-jmz} \quad . \quad . \quad . \quad . \quad (48)$$

For a current having the same phase and direction at the centre, but with the wave direction downwards, the current distribution would be

$$i' = I_0 e^{j(\omega t + \alpha)} e^{+jmz} \quad . \quad . \quad . \quad . \quad (49)$$

For the first, the components E_r and E_t are, as already shown in Section (4),

$$E_r = \frac{-30I}{r}\left\{\frac{r_2 - l}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (22)$$

$$E_t = \frac{30I}{r}\cot\theta\left\{\frac{r_2 - l + r/\cos\theta}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l + r/\cos\theta}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (23)$$

For the second, the same method of calculation as is outlined in Appendices 1 and 2, or, alternatively, the interchange of r_1 and r_2, substitution of $\theta + \pi$ for θ, and reversal of the sign of I, gives

$$E'_r = \frac{-30I}{r}\left\{\frac{r_2 + l}{r_2}e^{-jm(r_2-l)} - \frac{r_1 - l}{r_1}e^{-jm(r_1+l)}\right\} \quad . \quad (50)$$

and

$$E'_t = \frac{30I\cot\theta}{r}\left\{\frac{r_2 + l - r/\cos\theta}{r_2}e^{-jm(r_2-l)} - \frac{r_1 - l - r/\cos\theta}{r_1}e^{-jm(r_1+l)}\right\} \quad . \quad (51)$$

(10) THE SYNTHESIS OF A STANDING-WAVE RADIATOR

If both the forward and reversed wave [(48) and (49)] are carried simultaneously by the conductor, the total current at the point z is given by

$$i + i' = I_0 e^{j(\omega t+\alpha)}\left\{e^{-jmz} + e^{jmz}\right\} \quad . \quad . \quad (52)$$

$$= 2I_0 e^{j(\omega t+\alpha)} \cos mz \quad . \quad . \quad . \quad . \quad (53)$$

which is the exponential form of expression for a standing wave of maximum amplitude $2I_0$ at the centre and $2I_0 \cos ml$ at the ends. If in addition $\cos ml = 0$, i.e. for example, $2l = \lambda/2$, we have a half-wave dipole with zero current at the ends, i.e. a physically possible system. Thus the fields for a half-wave dipole can be derived by superposition of (22) and (50), and (23) and (51). Thus, for a current distribution on a half-wave dipole (i.e. $2l = \lambda/2$)

$$I_0 e^{j(\omega t+\alpha)} \cos mz \quad . \quad . \quad . \quad . \quad . \quad (54)$$

we have

$$\frac{E_r + E'_r}{2} = 30I \cdot \frac{l}{r} \cdot j\left(\frac{e^{-jmr_2}}{r_2} - \frac{e^{-jmr_1}}{r_1}\right) \quad . \quad (55)$$

and

$$\frac{E_t + E'_t}{2} = -30I \cdot \frac{\cot\theta}{r} \cdot j\left\{l\left(\frac{e^{-jmr_2}}{r_2} - \frac{e^{-jmr_1}}{r_1}\right) + \frac{r}{\cos\theta}\left(\frac{e^{-jmr_2}}{r_2} + \frac{e^{-jmr_1}}{r_1}\right)\right\} \quad . \quad (56)$$

It is easily shown that for distant points, and neglecting terms in $1/r^2$, the radial component vanishes and the tangential component is

$$-\frac{\cos(\pi/2\cos\theta)}{\sin\theta} \cdot \frac{e^{-jmr}}{r} 60I \quad . \quad . \quad . \quad (57)$$

Axial points need separate consideration, as before, and are found to give zero fields to the order $1/r^2$.

Other standing-wave distributions can, of course, be obtained by suitable combinations of the component progressive waves, as illustrated in this example. The process is not merely academic. The aerial system described in Ladner's paper* is one example of its practical application.

(11) APPLICATION OF FORMULAE TO A CLOSED-CIRCUIT RADIATOR

As a simple example of the application of the linear radiator formulae of Section (5) to a closed-circuit structure, consider the field at a distant point P due to the rectangular progressive-wave radiator shown in Fig. 2(*b*).

Let the current at A, the point of excitation, be

$$I = I_0 e^{j(\omega t+\alpha)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

Let I_n be the current at the point O_n. In the formulae of Section (5), the direction of the travel of the wave along the radiator is taken as the positive direction. With this convention, we have

$$\left.\begin{aligned} I_1 &= -I_6 = e^{-jmd/2}I \\ I_2 &= -I_5 = e^{-jm(l+d)}I \\ I_3 &= -I_4 = e^{-jm(2l+3d/2)}I \end{aligned}\right\} \quad . \quad . \quad (59)$$

Let $(E_t)_n$ be the tangential component of E due to the linear element with centre O_n. Then, from Section (8), only the vertical elements will contribute to E_t and P, and, from (32),

$$(E_t)_1 = -\sin ml \frac{e^{-jm(r-d)}}{r-d} e^{-jm(l+d)} 60jI \quad . \quad . \quad (60)$$

and

$$(E_t)_5 = \sin ml \frac{e^{-jm(r+d)}}{r+d} e^{-jm(l+d)} 60jI \quad . \quad . \quad (61)$$

By the simple addition of these terms, and neglecting terms in $1/r^2$ compared with terms in $1/r$,

$$E_t = \frac{\sin ml \sin md}{r} \cdot e^{-jm(l+d)} e^{-jmr} 120I \quad . \quad (62)$$

or, considering amplitudes only,

$$\hat{E}_t = \frac{120 \sin ml \sin md}{r} \hat{I} \quad . \quad . \quad . \quad (63)$$

Consider now the radial components. From (31)

$$(E_r)_2 = \sin ml \frac{e^{-jm(r-d)}}{r-d} e^{-jm(l+d)} 60jI \quad . \quad . \quad (64)$$

$$(E_r)_5 = -\sin ml \frac{e^{-jm(r+d)}}{r+d} e^{-jm(l+d)} 60jI \quad . \quad (65)$$

and, neglecting terms in $1/r_2$,

$$(E_r)_2 + (E_r)_5 = \frac{-\sin ml \sin md}{r} \cdot e^{-jm(l+d)} \cdot e^{-jmr} 120I \quad (66)$$

However, as pointed out in Section (7), the elements AF and CD are in a back-end-on relation to P, and each will contribute a radial component of order $1/r$ to the field at P. From (45)

$$(E_r)_6 = -\sin md \frac{e^{-jm(r+d/2)}}{r+d/2} e^{-jmd/2} 60jI \quad . \quad . \quad (67)$$

and

$$(E_r)_3 = \sin md \frac{e^{-jm(r-d/2)}}{r-d/2} e^{-jm(2l+3d/2)} 60jI \quad . \quad (68)$$

Therefore

$$(E_r)_6 + (E_r)_3 = \frac{\sin md \sin ml}{r} e^{-jm(l+d)} e^{-jmr} 120I \quad . \quad (69)$$

(neglecting terms in $1/r^2$ as before). Thus the radial components from the end-on elements just exactly cancel those from the vertical members, and the resultant field is wholly tangential and in the plane of the wave-front.

It is not intended in the present paper to multiply instances or to consider the complicated question of the actual design of structures for given types of directivity. This subject has already been dealt with in various more comprehensive papers, e.g. that by Foster.* The present paper is concerned more with certain distinctive features of the radiation from progressive-wave systems, and the above elementary example is included only to bring out the rather interesting point about the asymmetry of such radiators, particularly in relation to axial points.

A practical point of some interest, however, is the comparison between similar conductors carrying standing and progressive waves respectively, from the point of view of radiation resistance. This is considered in the next Section.

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* See Bibliography, (11).

* See Bibliography, (2).

(12) THE RADIATION RESISTANCE OF A LINEAR PROGRESSIVE-WAVE RADIATOR

From the formula in (26) for the component of the electric field parallel to the radiator, it is a comparatively simple matter to determine the self-radiation resistance of a linear progressive-wave radiator forming part of a closed circuit by the method of Pistolkors.* The evaluation is given in outline only since the theoretical basis of the method is very lucidly described in Pistolkors's paper.

For the current distribution

$$i = I_0 e^{j(\omega t+\alpha)} e^{-jmz}$$

$$= I e^{-jmz} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

in the radiator shown in Fig. 1, the field-component E_z, parallel to the radiator at the point P is, as shown in Section (4),

$$E_z = 30I\left\{\frac{e^{-jm(r_2+l)}}{r_2} - \frac{e^{-jm(r_1-l)}}{r_1}\right\} \quad . \quad . \quad (71)$$

and if the point P is situated on the radiator itself

$$r_1 + r_2 = 2l \quad . \quad . \quad . \quad . \quad . \quad (72)$$

and

$$E_z = -30Ie^{-jml}\left\{\frac{e^{jm(r_1-2l)}}{r_1 - 2l} + \frac{e^{-jm(r_1-2l)}}{r_1}\right\} \quad . \quad (73)$$

Putting

$$\left.\begin{aligned} i &= (a_1 + jb_1)I \\ \text{and} \qquad E_z &= (a_2 + jb_2)I \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (74)$$

the mean power contribution dW for the element dz is

$$dW = (a_1a_2 + b_1b_2)\frac{I_0^2}{2}dz \quad . \quad . \quad . \quad (75)$$

From (74), in conjunction with (70) and (71),

$$\left.\begin{aligned} a_1 &= \cos mz \\ b_1 &= -\sin mz \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (76)$$

$$\begin{aligned} a_2 &= \frac{1}{2l - r_1}\cos m(r_1 - 3l) - \frac{1}{r_1}\cos m(r_1 - l) \\ b_2 &= \frac{1}{l - r_1}\sin m(r_1 - 3l) + \frac{1}{r_1}\sin m(r_1 - l) \quad . \quad (77) \end{aligned}$$

Therefore

$$W = 30\frac{I_0^2}{2}\int_{-l}^{l}\left\{\frac{\cos m(r_1 - 3l + z)}{2l - r_1} - \frac{\cos m(r_1 - l - z)}{r_1}\right\}dz \quad . \quad (78)$$

(Note: The temptation to put $r_1 = z$ must be avoided at this stage, since r_1 is an intrinsically positive quantity and z is not. The more complicated substitution

$$r_1 = |z|$$

is avoided by a shift of origin.)

Putting

$$x = z + l$$

and

$$r_1 = x$$

we have

$$W = 30\frac{I_0^2}{2}\int_0^{2l}\left(\frac{\cos 2m(2l - x)}{2l - x} - \frac{1}{x}\right)dx \quad . \quad . \quad (79)$$

This is not integrable as it stands, but putting

$$2l - x = y \quad . \quad . \quad . \quad . \quad . \quad (80)$$

in the first integrand only we have

$$\int_0^{2l}\frac{\cos 2m(2l - x)}{2l - x}dx = \int_0^{2l}\frac{\cos 2my}{y}dy \quad . \quad . \quad (81)$$

$$= \int_0^{2l}\frac{\cos 2mx}{x}dx \quad . \quad . \quad . \quad . \quad (82)$$

Therefore

$$W = 30\frac{I_0^2}{2}\int_0^{2l}\frac{\cos 2mx - 1}{x}dx \quad . \quad . \quad . \quad . \quad (83)$$

$$= -30\frac{I_0^2}{2}\int_0^{4ml}\frac{1 - \cos u}{u}du \quad . \quad . \quad . \quad . \quad (84)$$

$$= -30\frac{I^2}{2}F(4ml) = -30\frac{I_0^2}{2}F\left(\frac{8\pi l}{\lambda}\right) \quad . \quad . \quad (85)$$

where $$F(x) = 0{\cdot}577 + \log_e x - Ci(x) \quad . \quad . \quad (86)$$

the numeral $0{\cdot}577$ being Euler's constant.

Defining the radiation resistance R as $|W|/\frac{1}{2}I_0^2$

$$R = 30F\left(\frac{8\pi l}{\lambda}\right)$$

By the same method, it can be shown that the " self " radiation resistance of a length $2l$ of a conductor of unspecified total length having a standing-wave current distribution

$$i = I_0 \sin mx e^{j(\omega t+\alpha)} \quad . \quad . \quad . \quad . \quad (87)$$

x being measured upward from the bottom of the conductor, is given by

$$R = 30\left\{\frac{1}{2}\left(1 + \cos^2\frac{4\pi l}{\lambda}\right)F\left(\frac{8\pi l}{\lambda}\right) - \frac{1}{4}\sin\frac{8\pi l}{\lambda}Si\left(\frac{8\pi l}{\lambda}\right)\right\} \quad . \quad . \quad (88)$$

In this case $2l$ can only be the total length of the conductor if it is so related to λ as to give zero current at the top open end, i.e. if $2l$ is a whole number of half wavelengths. For all such values of $2l$

$$\frac{1}{2}\left(1 + \cos^2\frac{4\pi l}{\lambda}\right) = 1$$

and $$\sin\frac{8\pi l}{\lambda} = 0$$

and $$R = 30F\left(\frac{8\pi l}{\lambda}\right)$$

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* See Bibliography, (8).

R being in this case the total radiation resistance, since it refers to the whole length of the conductor.

Thus we have the interesting result that the radiation resistance of a straight conductor of length $2l$ in free space is the same whether the current distribution be a standing wave of a given maximum amplitude or a progressive wave of the same amplitude, the length $2l$ being consistent with a standing wave distribution, i.e. a whole number of half wavelengths. For example, if $2l$ is half a wavelength, we have

$$\begin{aligned} \text{Euler's constant} &= 0\cdot577 \\ \log_e 2\pi &= 1\cdot835 \\ -C_i(2\pi) &= 0\cdot028 \\ &\overline{\ 2\cdot440} \end{aligned}$$

and $$R = 30 \times 2\cdot440 = 73\cdot2$$

(13) ACKNOWLEDGMENTS

The author is indebted to Mr. W. Ross of the Radio Department of the National Physical Laboratory and to Mr. I. H. Cole of the Bawdsey Research Station for critical comment and suggestion. The work was carried out as part of the programme of the Radio Research Board and is published by permission of the Department of Scientific and Industrial Research.

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APPENDIX I

The integral

$$F = \int_{-l}^{l} \frac{e^{-jm(r+z)}}{r} dz \quad . \quad . \quad . \quad . \quad (89)$$

where $$r^2 = a^2 + (b - z)^2 \quad . \quad . \quad . \quad . \quad (90)$$

figures so frequently in calculations relating to aerials that it is worth while to include here an outline of its evaluation.

Referring to Fig. 1,

$$r = a/\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

$$b - z = a \cot\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

Therefore $$r + z = b + a \tan \tfrac{1}{2}\theta \quad . \quad . \quad . \quad . \quad (93)$$

also $$dz = r d\theta/\sin\theta \quad . \quad . \quad . \quad . \quad . \quad (94)$$

and $$F = e^{-jmb} \int_{\theta 1}^{\theta 2} \frac{e^{-jma \tan \frac{1}{2}\theta}}{\sin\theta} d\theta \quad . \quad . \quad (95)$$

and putting $$-ma \tan \tfrac{1}{2}\theta = u \quad . \quad . \quad . \quad . \quad (96)$$

$$d\theta/\sin\theta = du/u \quad . \quad . \quad . \quad . \quad (97)$$

Therefore $$F = e^{-jmb} \int_{u_1}^{u_2} \frac{e^{ju}}{u} du \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

$$= e^{-jmb}\left\{Ei(u_2) - Ei(u_1)\right\} \quad . \quad . \quad . \quad (99)$$

where $$Ei(x) = Ci(x) + jSi(x) \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

$$u_2 = -ma \tan \tfrac{1}{2}\theta_2 = -m(r_2 - b + l) \quad . \quad (101)$$

and $$u_1 = -ma \tan \tfrac{1}{2}\theta_1 = -m(r_1 - b - l) \quad . \quad (102)$$

APPENDIX 2

It was shown in Section (2) that the scalar potential ψ and the time differential of the vector potential are both expressible in terms of the integral F, i.e.

$$\psi = 30I \,.\, F \quad . \quad . \quad . \quad . \quad (103)$$

and $$\frac{1}{c}\frac{d\mathbf{A}}{dt} = jm30\mathbf{I} \,.\, F \quad . \quad . \quad . \quad . \quad (104)$$

where $\mathbf{I}$ is a vector of magnitude $I = I_0 e^{j(\omega t + \alpha)}$

Since $$E = -\operatorname{grad}\psi - \frac{1}{c}\frac{d\mathbf{A}}{dt} \quad . \quad . \quad . \quad (105)$$

we shall require the differential of F with respect to the co-ordinates of P.

Let η be any co-ordinate of P. Then it can be shown that

$$\frac{\partial F}{\partial \eta} = -jm\frac{\partial b}{\partial \eta} \cdot F + e^{-jmb}\left\{\frac{\partial u_2}{\partial \eta} \cdot \frac{e^{ju_2}}{u_2} - \frac{\partial u_1}{\partial \eta}\frac{e^{ju_1}}{u_1}\right\} \quad (106)$$

For example, suppose we wish to evaluate E_t, the tangential component of E. In this case the relevant co-ordinates of P are r and θ (the r_0 and θ_0 of Fig. 1, but with the suffix omitted for convenience, since no ambiguity is involved).

The component of $\frac{1}{c}\frac{d\mathbf{A}}{dt}$ is, from (104),

$$-\left(\frac{1}{c}\frac{d\mathbf{A}}{dt}\right)_t = -jm \,.\, 30F \,.\, I \sin\theta \quad . \quad (107)$$

and since the differential elements are dr and $-rd\theta$, the component of $-\text{grad}\,\psi$ is given by

$$-(\text{grad}\,\psi)_t = -30I(\text{grad}\,F)_t = -30I\left(-\frac{1}{r}\frac{\partial F}{\partial \theta}\right) \quad (108)$$

$$= 30I \cdot \frac{1}{r}\left\{-jmF\frac{\partial b}{\partial \theta} + e^{-jmb}\frac{e^{ju_2}}{u_2}\frac{\partial u_2}{\partial \theta} - \frac{e^{ju_1}}{u_1}\frac{\partial u_1}{\partial \theta}\right\} \quad (109)$$

Since

$$b = r\cos\theta$$

$$\frac{\partial b}{\partial \theta} = -r\sin\theta \quad . \quad . \quad . \quad . \quad (110)$$

and the first term of (109) merely cancels the contribution from the vector potential [see (107)].

Since

$$u_2 = -m(r_2 - b + l) \quad . \quad . \quad . \quad (111)$$

and

$$r_2^2 = r^2 + l^2 - 2rl\cos\theta$$
$$= r^2 + l^2 - 2lb \quad . \quad . \quad . \quad . \quad (112)$$

it can be shown that

$$\frac{1}{u_2}\frac{\partial u_2}{\partial \theta} = \frac{r\sin\theta}{r_2}\,\frac{r_2 + l}{r_2 + b + l} \quad . \quad . \quad (113)$$

and by using the relationship

$$a^2 = r^2\sin^2\theta = (r_2 - b + l)(r_2 + b - l) \quad . \quad (114)$$

$$\frac{1}{u_2}\frac{\partial u_2}{\partial \theta} = \frac{1}{r_2}\,\frac{r + (r_2 - l)\cos\theta}{\sin\theta}$$

Similarly

$$\frac{1}{u_1}\frac{\partial u_1}{\partial \theta} = \frac{1}{r_1}\,\frac{r + (r_1 + l)\cos\theta}{\sin\theta} \quad . \quad . \quad (115)$$

Therefore, inserting these in (109) and multiplying through by e^{-jmb}

$$E_t = \frac{30I\cot\theta}{r}\left\{\frac{r_2 - l + r/\cos\theta}{r_2}e^{-jm(r_2+l)} - \frac{r_1 + l + r/\cos\theta}{r_1}e^{-jm(r_1-l)}\right\} \quad . \quad (116)$$

The similar evaluation of E_r involves $\partial F/\partial r$. Since

$$\frac{\partial b}{\partial r} = \cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (117)$$

the contribution from the vector potential is cancelled out by part of that from the scalar potential as before, and further it can be shown that

$$\frac{1}{u_2}\frac{\partial u_2}{\partial r} = \frac{1}{r} \cdot \frac{r_2 - l}{r_2}$$

and

$$\frac{1}{u_1}\frac{\partial u_1}{\partial r} = \frac{1}{r} \cdot \frac{r_1 + l}{r_1} \quad . \quad . \quad . \quad (118)$$

whence

$$E_r = \frac{-30I}{r}\left\{\frac{r_2 - l}{r_2} \cdot e^{-jm(r_2+l)} - \frac{r_1 + l}{r_1}e^{-jm(r_1-l)}\right\} \quad (119)$$

The determination of the parallel and perpendicular components E_z and E_x follows similar lines and need not be given in detail.

APPENDIX 3

Referring to Fig. 1, suppose the current at the ends of the radiator to give rise to terminal charges q_1 and q_2,

where

$$\left.\begin{aligned} q_1 &= -\int i_1 dt \\ q_2 &= \int i_2 dt \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

Since

$$i = I_0 e^{j(\omega t + d)} e^{-jmz} \quad . \quad . \quad . \quad . \quad (121)$$

$$i_1 = I_0 e^{j(\omega t + d)} e^{jml} \quad . \quad . \quad . \quad . \quad (122)$$

$$q_1 = -\frac{1}{j\omega} I_0 e^{j(\omega t + d)} e^{jml} \quad . \quad . \quad . \quad (123)$$

and

$$[q_1] = -\frac{1}{j\omega} I_0 e^{j(\omega t + d)} e^{-jm(r_1 - l)}$$

$$= -\frac{1}{j\omega} I e^{-jm(r_1 - l)} \quad . \quad . \quad . \quad . \quad (124)$$

Similarly

$$[q_2] = \frac{1}{j\omega} I e^{-jm(r_2 + l)} \quad . \quad . \quad . \quad . \quad (125)$$

The field $\mathbf{E}_q$ at P due to these hypothetical charges is given by

$$\mathbf{E}_q = -\text{grad}\,\psi_1 - \text{grad}\,\psi_2 \quad . \quad . \quad (126)$$

where, converting to practical units,

$$\left.\begin{aligned} \psi_1 &= -30I\frac{1}{jm}\,\frac{e^{-jm(r_1 - l)}}{r_1} \\ \psi_2 &= 30I\frac{1}{jm}\,\frac{e^{-jm(r_2 + l)}}{r_2} \end{aligned}\right\} \quad . \quad . \quad (127)$$

Thus, for the radial component $(E_q)_r$ of the field at P due to these charges

$$(E_q)_r = -\frac{\partial \psi_1}{\partial r} - \frac{\partial \psi_2}{\partial r} \quad . \quad . \quad . \quad (128)$$

and since

$$r_2^2 = r^2 + l^2 - 2rl\cos\theta$$

and

$$r_1^2 = r^2 + l^2 + 2rl\cos\theta \quad . \quad . \quad . \quad (129)$$

$$\left.\begin{aligned} \frac{\partial r_2}{\partial r} &= \frac{r - l\cos\theta}{r_2} \\ \text{and} \qquad \frac{\partial r_1}{\partial r} &= \frac{r + l\cos\theta}{r_1} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (130)$$

Therefore

$$\left.\begin{aligned} -\frac{\partial\psi}{\partial r} &= -30I\frac{r + l\cos\theta}{r_1}\cdot\frac{e^{-jm(r_1-l)}}{r_1}\cdot\frac{(jm+1/r_1)}{jm} \\ \text{and} \\ -\frac{\partial\psi_2}{\partial r} &= \quad 30I\frac{r - l\cos\theta}{r_2}\cdot\frac{e^{-jm(r_2+l)}}{r_2}\cdot\frac{(jm+1/r_2)}{jm} \end{aligned}\right\} \quad (131)$$

and $(E_q)_r = -\left(\frac{\partial\psi_1}{\partial r} + \frac{\partial\psi_2}{\partial r}\right)$

$$= 30I\left\{1 - \frac{j}{mr_2}\left(\frac{r - l\cos\theta}{r_2}\right)\frac{e^{-jm(r_2+l)}}{r_2} - \left(1 - \frac{j}{mr_1}\right)\left(\frac{r + l\cos\theta}{r_1}\right)\frac{e^{-jm(r_1-l)}}{r_1}\right\} \quad (132)$$

A similar evaluation of $\frac{1}{r}\frac{\partial\psi}{\partial\theta}$ leads to

$$(E_q)_t = -30\ Il\sin\theta\left\{\frac{e^{-jm(r_2+l)}}{r_2^2} - \frac{e^{-jm(r_1-l)}}{r_1^2}\right\} \quad (133)$$

AN EXPERIMENTAL INVESTIGATION OF RESONANCE AND ELECTRONIC OSCILLATIONS IN MAGNETRONS*

By J. S. McPETRIE, Ph.D., D.Sc., and L. H. FORD, M.Sc., Associate Members.

(*Paper received* 26*th June*, 1939.)

SUMMARY

The paper describes an experimental investigation of the oscillations produced by magnetrons. It is shown that in all types of magnetron valves, oscillations are possible for which the ratio of the product of wavelength and anode voltage to magnetic field has a series of discrete values given by the series

$$K : K/2 : K/3 \ . \ . \ . \ K/n$$

in which K is a function of the anode diameter. Oscillations at frequencies corresponding to values of n up to 7 have been observed. In a full-anode magnetron all the oscillation intensities are small. For a 2-segment-anode magnetron the amplitude of the oscillation of lowest frequency ($n = 1$) is pronounced; in a 4-segment-anode magnetron oscillations at the frequency corresponding to $n = 2$ are predominant, and so on. These oscillations all belong to the class known as resonance oscillations. It is shown that there is no clear distinction between resonance and electronic oscillations at very short wavelengths, and it is suggested in the paper that they are essentially of the same nature.

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(1) INTRODUCTION

The magnetron oscillator as a source of high-frequency radiation has been known for many years, but there is still no completely satisfactory explanation of its very complex behaviour. The wavelength obtained is dependent on the form of the anode of the valve, the nature of the external circuit, the anode voltage applied, and the strength of the magnetic field used. Oscillations can be classified into a number of types according to the relationships which hold among these quantities. At present, at least three such types of oscillation are recognized in magnetrons.[1,2]

(A) Electronic Oscillations.

These oscillations occur near the so-called critical value of magnetic field where the anode current decreases rapidly with increase in magnetic field intensity. It has been found empirically that the product of wavelength (λ) in centimetres and magnetic field (H) in oersteds is a constant of about 11 000.[3,4,5] The frequency of electronic oscillations, in general, is higher than the fundamental resonant frequency of the smallest external circuit which can be attached to the valve, and the external circuit, which is usually in the form of Lecher wires, must be adjusted in length so that an overtone resonance coincides in frequency with the frequency of the oscillations. It is generally observed that the maximum amplitude of electronic oscillation occurs with relatively low filament emission.[6] Electronic oscillations have been found in single- and multi-anode segment magnetrons.

(B) Dynatron Oscillations.

Near the critical magnetic field in a split-anode magnetron a simple negative resistance condition is obtained suitable for the production of oscillations at the resonant frequency of any circuit connected across the anode segments. With a fixed anode voltage the optimum value of magnetic field is independent of the wavelength of oscillation.

(C) Resonance Oscillations.

These oscillations are similar to electronic oscillations in that the frequency is dependent on the operating conditions of the valve, but they are found not only near the critical magnetic field but at all values of magnetic field greater than the critical value. For a given valve the wavelength of resonance oscillations is proportional to the ratio of magnetic field to anode voltage. It has been suggested[2] that there is no clear line of demarcation between resonance and electronic oscillations.

Since the magnetron is primarily of interest as a source of power at very short wavelengths most investigators have concentrated their attention on this aspect of the problem. For very short wavelengths, however, all types of oscillations occur near the critical field, and much of the confusion into which the study of the magnetron has fallen is due to this fact. It is only when the wavelength is greater than about 5 metres that separation between the condition for the dynatron and resonance types of oscillation is possible. On these relatively long wavelengths no electronic oscillations are possible and the resonance oscillations occur at magnetic field intensities much greater than the critical field, at which the dynatron oscillations are still obtained.

In order to overcome the difficulties in differentiation between dynatron and resonance oscillations in multi-segment valves, and also to enable observations to be made on all the types of oscillations at approximately the same wavelength, the authors have made a number of measurements with the conditions adjusted so as to make the oscillation amplitude in these valves as small as possible. In this way the range of magnetic field (the most common variable) over which the oscillations occurred was limited. The measurements were made on valves with a full cylindrical anode and also with 2- and 4-segment anodes, and were directed towards determining the circumstances under which oscillations could be obtained rather than to finding the conditions for greatest efficiency.

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* Official communication from the National Physical Laboratory.

Reprinted from *Journal I.E.E.*, 1940, **86**, p. 283.

(2) EXPERIMENTAL PROCEDURE AND RESULTS

(a) Full-anode Valve

The dynatron type of oscillation is not possible with a magnetron valve which has a single cylindrical anode, and it is generally assumed that only oscillations of the electronic type occurring near the critical field and satisfying the λH = constant law can be obtained with such valves.[7] Experiments made at the National Physical Laboratory with a full-anode valve showed, however, that oscillations could be produced with magnetic field intensities much greater than the critical value and with various values of λH. There is considerable difficulty in devising a satisfactory external circuit for a full-anode valve. A Lecher wire system connected to anode and cathode and terminated by a capacitance of a few micro-microfarads was tried, but it was found that both the wavelength and amplitude of the oscillations were independent of the length of the circuit. No observable difference was obtained by varying the positions on the circuit of the points at which the H.T. supply to the magnetron was connected.

The anode voltage in each series of tests was kept constant, and the magnetic field, provided by an electromagnet, adjusted until oscillations were detected by means of a diode rectifier coupled closely to the circuit. According to the accepted hypothesis for the production of oscillations in single-anode-segment magnetrons these oscillations occur only near the critical magnetic field and their wavelength is such that

$$\lambda H = 11\,000 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It was found,* however, that for each anode voltage oscillations could be obtained from about the critical magnetic field up to the greatest magnetic field available and that the wavelength, in any particular case, did not necessarily obey relation (1). For the same magnetic field, oscillations at two or even three wavelengths were sometimes present simultaneously, the wavelengths bearing usually some simple relation to one another, such as 1 : 2, 2 : 3. Results are shown in Fig. 1 in which for an anode voltage of 540 the wavelengths of the oscillations are plotted as ordinates against the corresponding values of magnetic field as abscissae. It will be seen that the oscillations occur at magnetic fields much greater than the critical value shown by the vertical dotted line in the figure, and that the experimental points lie closely to a series of parallel lines given by

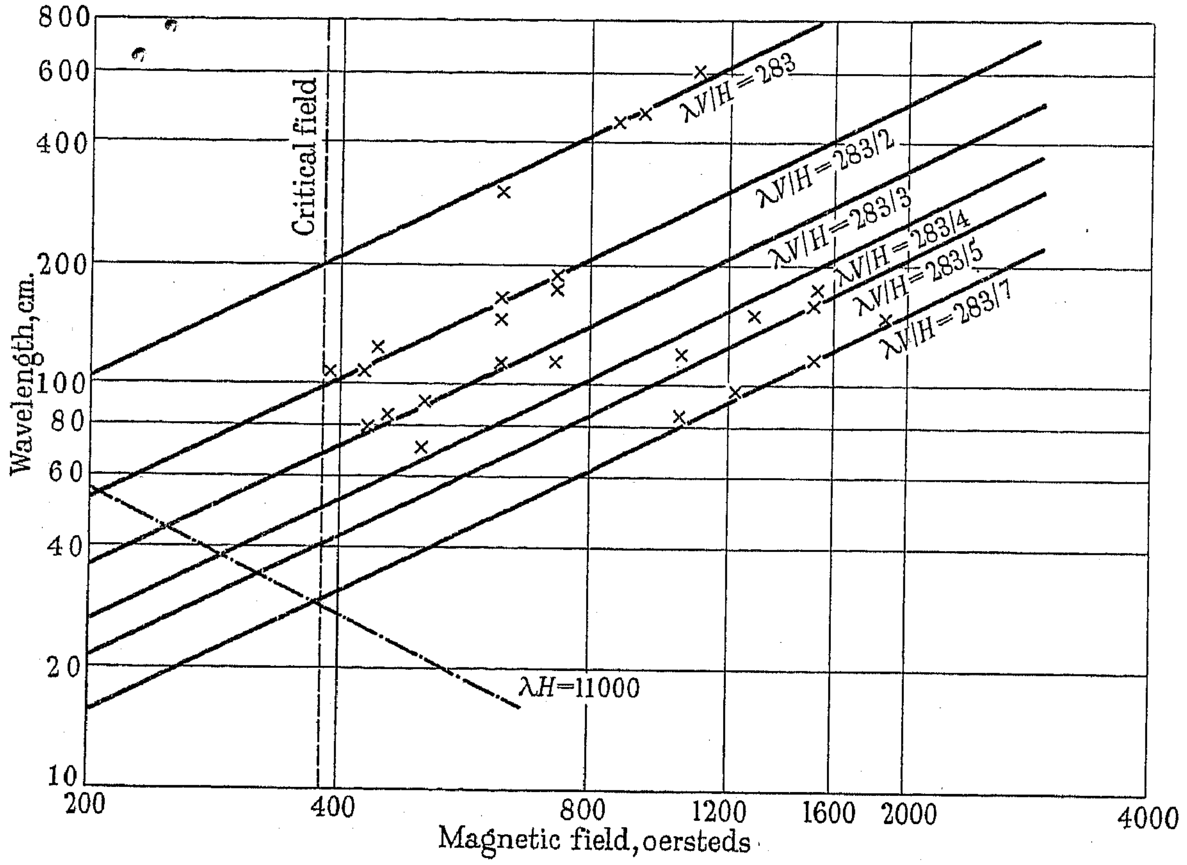

Fig. 1.—Wavelength of oscillations of full-anode magnetron.
$V_a = 540$. Anode diameter = 1 cm. (nominal).

$$\frac{\lambda}{H} = \frac{\text{const.}}{n} : n = 1, 2, 3, \text{etc.} \quad . \quad . \quad . \quad (2)$$

As mentioned above, oscillations at two or three frequencies were obtained, but usually the different oscillations occurring at one magnetic field intensity could be differentiated by suitable adjustment of filament emission. For example, with a magnetic field intensity of 1 040 oersteds an oscillation at a wavelength of 83 cm. was observed with full filament current of 6 amperes; when the filament current was reduced to 5·7 amperes this oscillation disappeared and was replaced by an oscillation at 118 cm. wavelength. On further reduction of filament current, this oscillation ceased and, with only slight readjustment of magnetic field, oscillations at 610 cm. were obtained.

A point of considerable interest is that although the most careful search failed to detect any normal electronic oscillations complying with the condition $\lambda H = 11\,000$ near the critical value of magnetic field, the line given by relation (2) with $n = 7$ cuts the line corresponding to the critical field at the same point as the broken line in Fig. 1 and given by relation (1). This result was not peculiar to this experiment. Although anode voltages ranging from 120 to 600 were used, in no case were electronic oscillations of the usual type observed. In all cases, however, where a sufficient number of oscillating conditions were obtained to enable a series of lines to be drawn as in Fig. 1, the line given by $n = 7$ cut that corresponding to the critical field at the point at which electronic oscillations should have appeared according to other observers.[7]

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* It is of interest to note that Okabe[8] states he has also found oscillations at a magnetic field greater than the critical value. He gives few details, however, and only one experimental result.

Results similar to those shown in Fig. 1 were obtained with other anode voltages, the positions of the parallel lines altering in such a way that for a given wavelength the ratio of anode voltage to magnetic field intensity remained constant. The slope of the lines in Fig. 1 is such that for each line λ/H = constant, so that as H for a given wavelength λ is proportional to the anode voltage V, $\lambda V/H$ = constant. The values of $\lambda V/H$ for a series of anode voltages are given in Table 1; they correspond to the lines having respectively the maximum and minimum observed values of $\lambda V/H$.

Table 1

Anode voltage	λ	Magnetic field	$\frac{\lambda V}{H}$
	cm.	oersteds	
300	590	560	315
	140	1 040	40
360	500	640	280
	98	840	42
480	480	820	280
	145	1 580	44
600	580	1 120	310
	115	1 600	43

The average maximum and minimum values of $\lambda V/H$ from this Table are respectively 296 and 42. The ratio of these two numbers is about 7, showing that for a given magnetic field there was a ratio of 7 in the range of wavelengths of the oscillations obtainable.

(b) Two-Segment Magnetron

Both dynatron and resonance oscillations can readily be obtained with a two-segment magnetron at relatively long wavelengths (5 metres and upwards), but at lower wavelengths, as pointed out above, it is difficult to differentiate between the two types. It has been shown[1] that the relation between the wavelength λ (cm.) of the resonance oscillations, the magnetic intensity (H) in oersteds, and anode voltage (V) is given by

$$\frac{\lambda V}{pd^2H} = 430 \qquad (3)$$

in which d is the diameter in cm. of the anode and p the number of pairs of anode segments (equal to unity in a 2-segment valve). The experimental fact that the operating conditions must be adjusted for each particular wavelength suggests that there must be some form of resonance within the valve, whence the name applied to the oscillations. Experimental evidence of this resonance has been given,[2] and an equivalent electrical circuit deduced for a magnetron in the neighbourhood of these resonance oscillations. The resonance found in magnetrons is similar to that observed in the case of an electronic oscillator of the positive-grid triode type. The electronic oscillations obtained near the critical magnetic field in a magnetron have the same characteristics as those found in the positive-grid triode, both being due to pronounced inter-electrode transit time effects. The critical magnetic field of a magnetron having an anode diameter of d cm. is given by

$$\frac{d^2H^2}{V} = 180 \qquad (4)$$

This relation, combined with the empirical result λH = 11 000, can be combined to yield the relation

$$\frac{\lambda V}{d^2H} = 61 \qquad (5)$$

It will be noted that relation (5) is of exactly the same form as relation (3) for resonance oscillations when p in the latter expression is made equal to unity, but that the ratio of the values of the two factors is almost exactly 7. It is interesting to note in passing that this number represents the range in value of $\lambda V/H$ obtained with the full-anode magnetron and given in Table 1. If, as suggested,[2] resonance and electronic oscillations are essentially of the same form some explanation of the apparent discrepancy between the two experimental results represented by relations (3) and (5) is required. The chief distinction between resonance and electronic oscillations in any magnetron for a given anode voltage is the wavelength of the oscillations, the resonance wavelength being, in general, many times greater than the electronic. The wavelengths used by Gill and Britton[1] to determine relation (3) were of the order of 40 metres, while the wavelength of the electronic oscillations from which relations (1) and (5) have been derived has usually been of the order of 20 to 30 cm. Experiments were made, therefore, to determine whether the constancy of $\lambda V/H$ observed in the previous measurements at the longer wavelengths was maintained over the whole possible range of wavelengths, or was, in fact, a function of the wavelength of the oscillations. If the latter result was obtained, then in view of the previously shown similarity between resonance and electronic oscillations they could well be the same type of oscillation occurring at different wavelengths.

A 2-segment magnetron was connected to an external circuit consisting of a variable condenser and a series of plug-in coils. The wave-range covered in this way was from about 1·5 to 40 metres. The experimental procedure was to set the circuit to the desired wavelength and to vary the magnetic field without alteration of anode voltage until oscillations were detected by means of a diode rectifier coupled closely to the magnetron. The magnetic field was then adjusted until the oscillation intensity was a maximum, and the filament current was reduced until the oscillations could just be detected. These two adjustments were not, in general, completely independent of one another and the procedure had to be repeated several times before the optimum field at the lowest possible filament current could be determined. The filament current was reduced in this way so as to limit the amplitude of the oscillations and thus make it possible to differentiate between the different forms of oscillation by preventing any overlapping between them.

The results of a series of measurements made in this way are shown in Fig. 2, from which it will be seen that the results are somewhat complex. The straight line A corresponds to the dynatron oscillations occurring near the critical magnetic field. Since this field is dependent only on the anode voltage applied to the valve and not on the wavelength of the oscillations, one would expect, as shown experimentally, the points to lie on a straight line passing through the origin. In addition to the line A a number of other straight lines radiating from the origin were obtained. These lines represent oscillations which occur at certain values of magnetic field greater than the critical value but which appear to be of the dynatron type since the wavelength corresponds to the fundamental frequency of the external circuit, and the magnetic field at which they occur is independent of the wavelength. The resonance oscillations are shown by the curve R from which it is evident that although the value of $\lambda V/H$ is a constant for the longer wavelengths it decreases gradually for low values. At the shortest wavelength it was difficult to differentiate between dynatron and resonance oscillations, since a very small change in the current in the magnetizing coils was sufficient to alter the oscillating condition from one form to the other. By keeping the filament current small, however, it was usually possible to distinguish two peaks of oscillation output as the magnetic field intensity was changed. One maximum output condition corresponded to the dynatron oscillations on line A and the other to the resonance oscillations on curve R. If the amplitude of oscillation was large these two maximum output conditions merged into one at an intermediate value of magnetic field. There appears to be no doubt, however, that the value of $\lambda V/H$ for resonance oscillations with a given valve does decrease with decrease in wavelength.

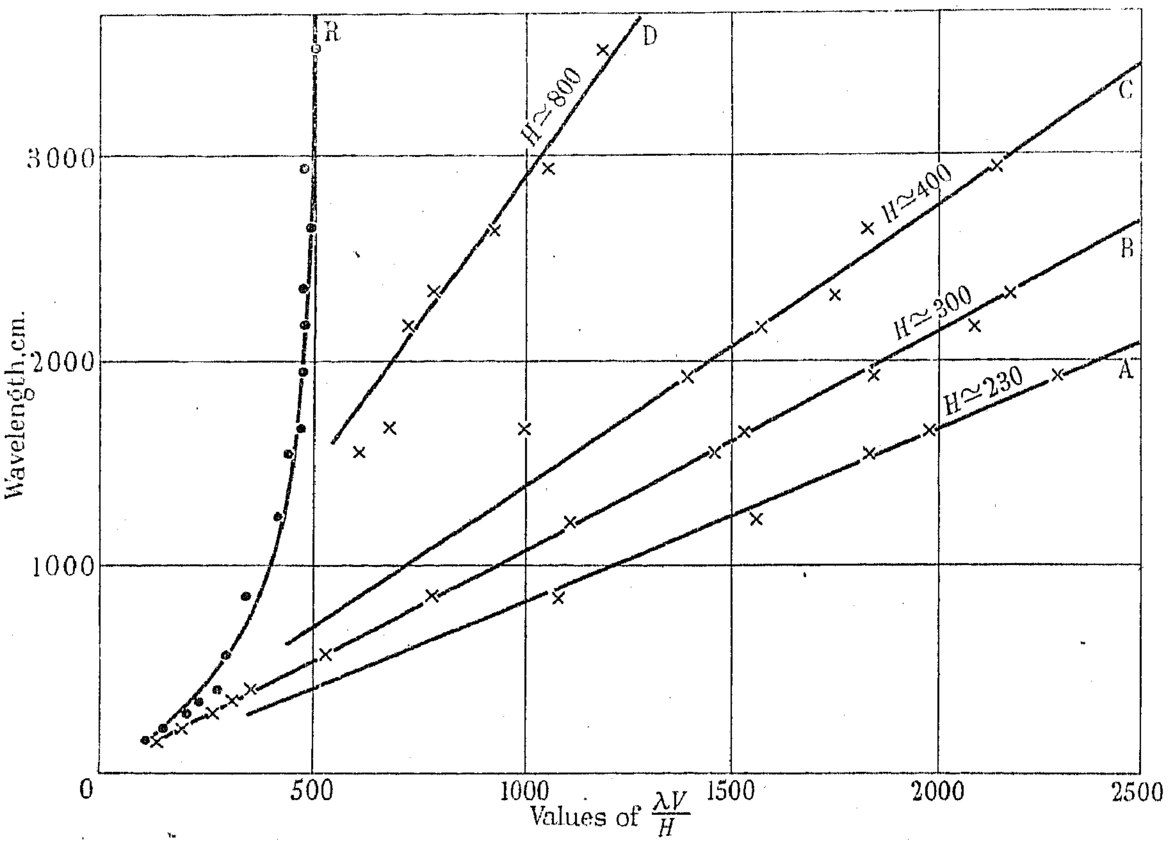

Fig. 2.—Wavelength plotted against $\lambda V/H$ for 2-segment anode magnetron.
$V = 300$ volts. λ in cm., V in volts, H in oersteds.

Results similar to those plotted in Fig. 2 have been obtained with a number of anode voltages and with various angles between the valve axis and the direction of the magnetic field. In each case the same general type of curve was obtained, the effect of an increase in tilt being to lessen somewhat the curvature of the $\lambda/(\lambda V/H)$ curve at short wavelengths.

In Fig. 2 the magnetic fields required for dynatron and resonance oscillations are shown; experiments were also made to determine whether, in addition to these, unclassified oscillations could be obtained at any wavelength at a magnetic field greater than that required for resonance oscillations at that wavelength. Such oscillations were found, although their amplitude was in general extremely small. The wavelengths of the oscillations are shown in Fig. 3 plotted as ordinates against the corresponding magnetic fields as abscissae for an anode voltage of 600. The curve for the usual type of resonance oscillations again shows a decrease of $\lambda V/H$ with diminishing wavelength for small values of the latter. Lines, similar to those shown in Fig. 1 for the single-anode valve corresponding to values of $\lambda V/H$ equal to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{7}$ of the final magnitude of that quantity for long-wavelength resonance oscillations, have been drawn in the Figure. It will be seen that there is a distinct tendency for the oscillations to lie on these lines. It is of special interest to note that the electronic oscillations ($\lambda H = 11\,000$) occurred on the one-seventh line. This raises the possibility that for these weak oscillations, of which the electronic may be a special case, the 2-segment valve behaves in the same manner as a full-anode valve. The values of $\lambda V/H$ for the top lines in Figs. 1 and 3 are respectively 280 and 450. The valves had each a nominal diameter of 1 cm., but the measured critical magnetic fields were such that the ratio of the effective diameters was actually 1·18. Assuming in accordance with (3) that $\lambda V/H$ is proportional to the square of the anode diameters, $(450/280)^{\frac{1}{2}}$ should be equal to 1·18. Its actual value is 1·27, which is near enough to 1·18 to give support to the hypothesis that the series of values of $\lambda V/H$ in both full-anode and 2-segment-anode valves are identical for valves of the same dimensions.

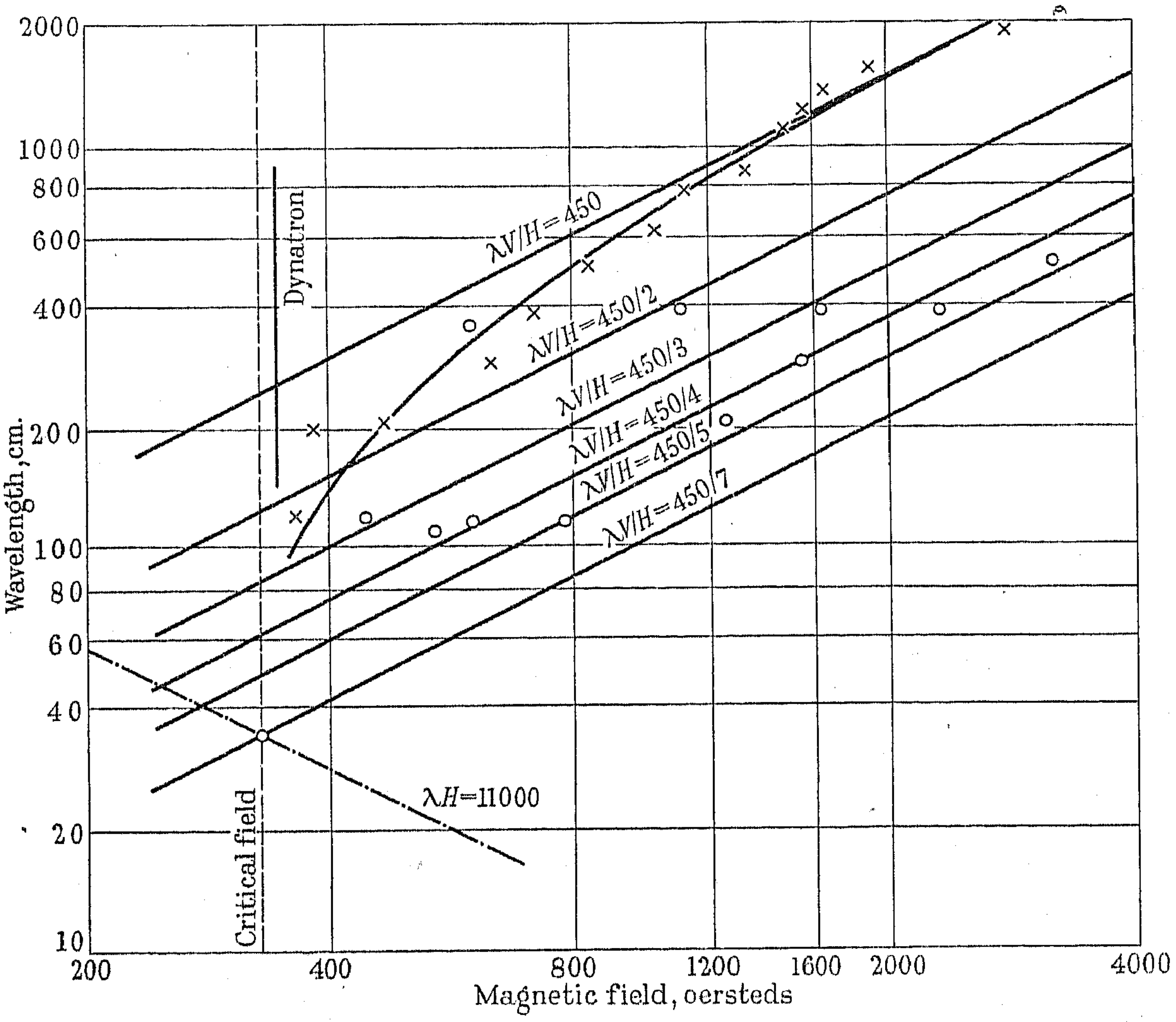

Fig. 3.—Wavelength of oscillations of 2-segment-anode magnetron.

V_a = 600. Anode diameter = 1 cm. (nominal).
× × Strong oscillations.
○ ○ Weak oscillations.

The results shown in Figs. 2 and 3 establish that the value of $\lambda V/H$ for resonance oscillations, which is constant for wavelengths greater than 5 metres, decreases with wavelength down to the lower limit of the latter determined by the smallest circuit which can be connected to the valve. A further comprehensive series of tests was made on a 2-segment-anode magnetron in order to determine its behaviour at still shorter wavelengths with a special view to finding the relation, if any, between resonance and electronic oscillations. For this purpose a Lecher wire circuit was used as the external circuit and the conditions for oscillation were noted with various lengths of circuit at a series of anode voltages between 60 and 2 000.

The experimental results may conveniently be summarized as follows:—

(1) The value of λH was found to vary considerably, in general over a range 8 000 to 14 000, with a preponderance of values close to 11 000, especially at high anode potentials. In addition, a few scattered values of about 6 000 were noted at low voltages as reported by Okabe,[9] and values of 22 000 also occurred.

(2) The wavelength with any particular valve did not vary smoothly with anode voltage. Certain definite wavelengths would appear with two or three different values of anode voltage and then be replaced by other widely different wavelengths. This effect has been observed also by other workers.[3]

(3) Oscillations obeying the electronic law $\lambda H = 11\,000$ were not confined exactly to the critical magnetic field but occurred at field intensities both greater and smaller than that field. At high anode voltage the electronic oscillations occurred usually at magnetic fields about 20 % in excess of the critical field.

A list of observations made with a 2-segment 1 cm. diameter anode valve, illustrating some of the points enumerated above, is given in Table 2. The results given in the Table have been so selected that with one or two exceptions the product of wavelength and magnetic field was very roughly equal to 11 000.

In all the above results the value of the observed magnetic field which corresponded to maximum amplitude of oscillation was subject to an error of about 10 %. The filament current was, in general, considerably below that for full emission, and showed a fairly pronounced optimum value for each oscillation.

Most of the oscillations recorded in Table 2 are such that the value of λH is so nearly 11 000 that they would be classed as electronic oscillations. It will be noted, however, that the magnetic field at which they occurred was sometimes less and sometimes greater than the critical field calculated from the diameter (1 cm.) of the anode. A number of static characteristics of anode current against magnetic field were taken and these showed that the magnetic field calculated from relation (4), viz. $H^2 = 180V$, always lay on the steep portion of that characteristic and was a reliable indication of the critical field. Certain of the oscillations given in Table 2 occurred, therefore, at magnetic fields appreciably different from the critical value. In Fig. 4 the value of $\lambda V/H$ given in Table 2 is plotted against the ratio of the actual to critical magnetic fields. It will be seen that there is a continuous variation in the magnitude of $\lambda V/H$, and that when the actual field equals the critical field the value of $\lambda V/H$ is approximately 60 as noted in relation (5).

Table 2

Anode volts V	Wave-length λ, cm.	Critical magnetic field $[H_c = \sqrt{(180V)}]$	Observed field H	λH	$\frac{\lambda V}{H}$
60	180	105	60	11 000	180
	67		82	5 500	49
120	136	145	95	13 000	170
	88		105	9 300	100
	63		170	10 700	44
180	64	180	160	10 200	72
	38		190	7 200	36
240	60	210	186	11 000	77
	39		270	10 500	35
300	65	230	210	14 000	93
	39		270	10 500	43
480	29	295	370	11 000	38
600	29	330	370	11 000	47
1 100	21	445	540	11 500	43
1 500	19·5	520	600	12 000	49
	15·5		700	11 000	33
1 800	14·5	570	780	11 500	34

The tendency for electronic oscillations to be confined to a limited number of wavelengths is brought out in Table 3, in which the observed wavelengths recorded in Table 2 are given in the first column.

The Table also shows that the ratio of any two consecutive wavelengths was approximately equal to $\sqrt{2}$. The practical importance of the results emphasized in Table 3 is that electronic oscillations tend to occur at discrete wavelengths, and that no adjustment of magnetic field or anode voltage will enable a continuous adjustment of wavelength to be made over any considerable range. It should be pointed out that this discontinuity in wavelengths was obtained only for wavelengths so short that the external circuit was not resonant at its fundamental frequency. The series of wavelengths of oscillations given in Table 3 should not be confused with the so-called higher-order electronic oscillations reported by Groos[5] and other workers. The product of wavelength and magnetic field for these oscillations in a

Table 3

Observed wavelength, cm.	S	$15 \times 2^{S/2}$
15	0	15
21	1	21
29	2	30
39	3	42
62	4	60
88	5	85
136	6	120
180	7	170

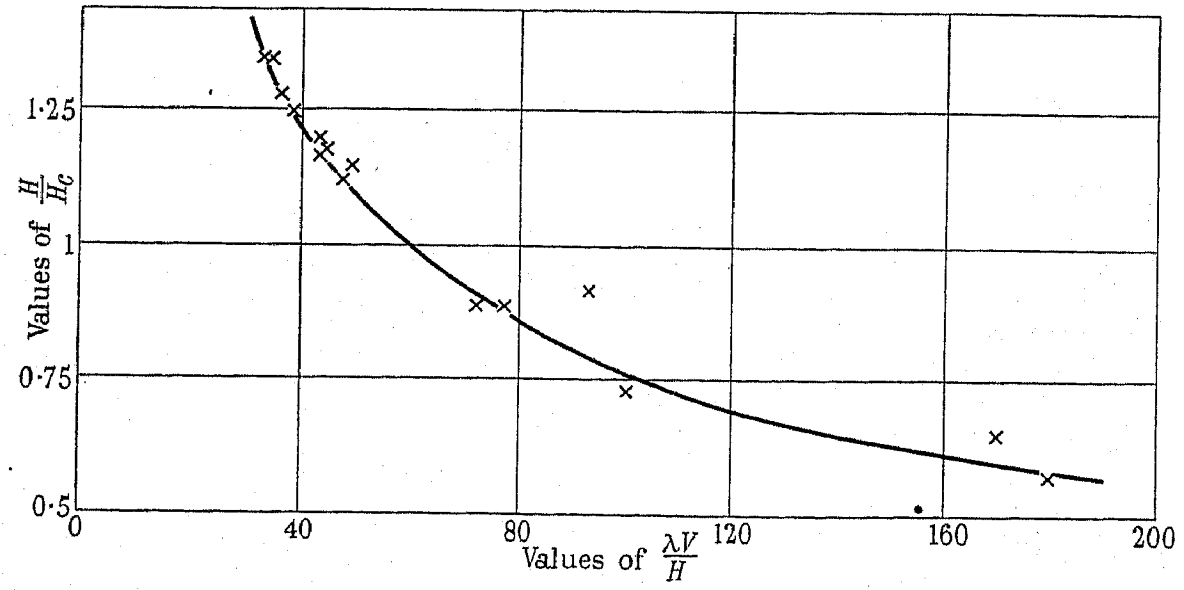

Fig. 4.—Ratio of actual H to critical H, plotted against $\lambda V/H$ for electronic oscillations. $\lambda H \simeq 11\,000$ for 2-segment-anode magnetron with anode voltages 60–1 800.

2-segment-anode magnetron has a series of discrete values greater than 11 000. The oscillations given in Table 2 were obtained for a wide range in anode voltages and magnetic fields but were all such that the product of wavelength and magnetic field intensity was approximately 11 000. The higher-order electronic oscillations, on the other hand, are such that that product has a series of values. These higher-order electronic oscillations were observed but they did not agree closely with the formulae given by Groos.

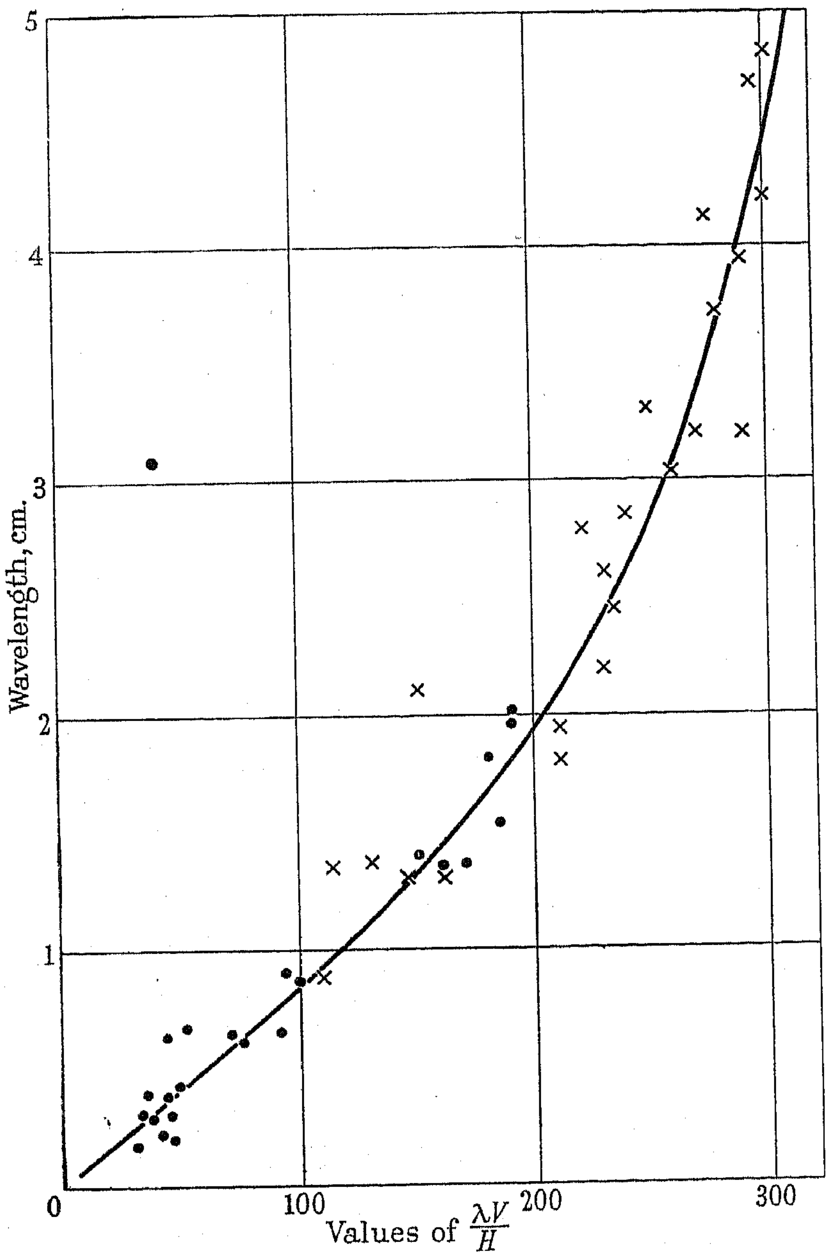

Fig. 5.—Wavelength plotted against $\lambda V/H$ for 2-segment-anode magnetron: resonance and electronic oscillations.

V_a = 60–1 800; λ in cm., V in volts, H in oersteds.
× × Resonance oscillations.
● ● Electronic oscillations ($\lambda H \simeq 11\,000$).

Electronic oscillations of longer wavelength fall into the region in which resonance oscillations occur. This is shown in Fig. 5, in which the value of $\lambda V/H$ is plotted against λ for a large number of oscillations obtained with a 2-segment valve at a series of anode voltages varying from 60 to 2 000. The resonance oscillations are shown by crosses, while those oscillations for which $\lambda H = 11\,000 \pm 3\,000$ are shown by dots. It will be seen that both the sets of points lie on a smooth curve and that they overlap in wavelength. The dots represent oscillations which would be termed electronic, while the crosses correspond to resonance oscillations. For a 2-segment valve similar in dimensions to that used in the above experiments Gill and Britton determined the magnitude of $\lambda V/H$ to be approximately 400 for resonance oscillations. As the value of $\lambda V/H$ in Fig. 5 decreases continuously with decrease in wavelength from about 400 to 30–40 for oscillations which obey the electronic law $\lambda H = 11\,000$, there would appear from the experimental results to be no essential difference between these two types of oscillation.

(c) Four-Segment Magnetron

A number of measurements similar to those described above were made with a 4-segment-anode valve having the same nominal dimensions as the full-anode and 2-segment-anode valves used in the earlier experiments. As is common practice with multi-segment valves, alternate segments were connected together within the valve envelope. With the 4-segment valve the tendency to produce dynatron oscillations was much less pronounced than with the 2-segment valve. This characteristic of the 4-segment valve allowed a fuller investigation of the conditions required for the production of short-wave resonance oscillations. This could not be done so easily with the 2-segment valve, in which dynatron and resonance oscillations of short wavelength occurred under closely similar conditions. A Lecher wire circuit was used in this case so that overtone resonances of the external circuit would allow of very short-wavelength oscillations being produced. The results obtained with the 4-segment

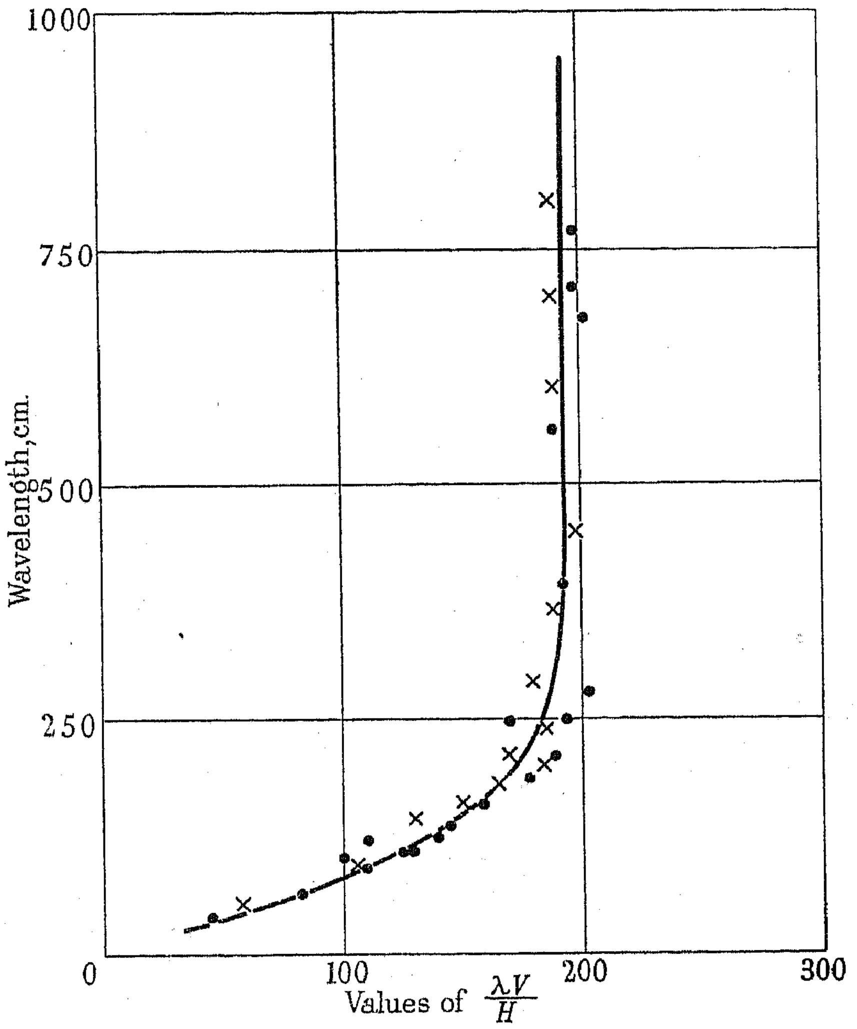

Fig. 6.—Wavelength plotted against $\lambda V/H$ for 4-segment-anode magnetron. λ in cm., V in volts, H in oersteds.

× × V = 180 volts.
● ● V = 300 volts.

valve are given in Fig. 6. Considerable difficulty was experienced at the longer wavelengths owing to the variation of the optimum value of magnetic field for a given wavelength with the setting of filament current. An increase in filament current produced at least an equal percentage increase in the value of the optimum magnetic field. Since the minimum filament current for which oscillations could be maintained tended to increase with wavelength, this resulted in a distinct tendency for the observed value of $\lambda V/H$ to decrease with increase in wavelength above about 10 metres. The results given in Fig. 6 were obtained with the oscillation amplitude as small as possible. They are similar to those found for the similar to those described above on a 2-segment valve, to see whether oscillations other than the normal dynatron or resonance types could be obtained. The results of these experiments are shown plotted in Fig. 7. It will be seen that although most of the points lie near the curve which coincides asymptotically with the straight line given by $\lambda V/H = 190$, a number of oscillating conditions were obtained for which the value of $\lambda V/H$ was approximately 400. This latter value is similar to that found in the case of the 2-segment valve and also, in fact, coincides with the maximum value of $\lambda V/H$ obtainable with the full-anode valve when correction is made for the difference in apparent diameters of the two anodes. The dots shown in Fig. 7 correspond to weak oscillations. It will be seen that they lie approximately on the lines given by $\lambda V/H = 380/n$. These values of $\lambda V/H$ are the same as those found for both 2- and single-segment valves of the same anode diameter. The results suggest, therefore, that both the 4- and 2-segment valves may act as single as well as multi-segment valves. This is not surprising when it is remembered that in the case of the full cylindrical-anode valve the external circuit appeared to have little or no effect on the wavelength of oscillation.

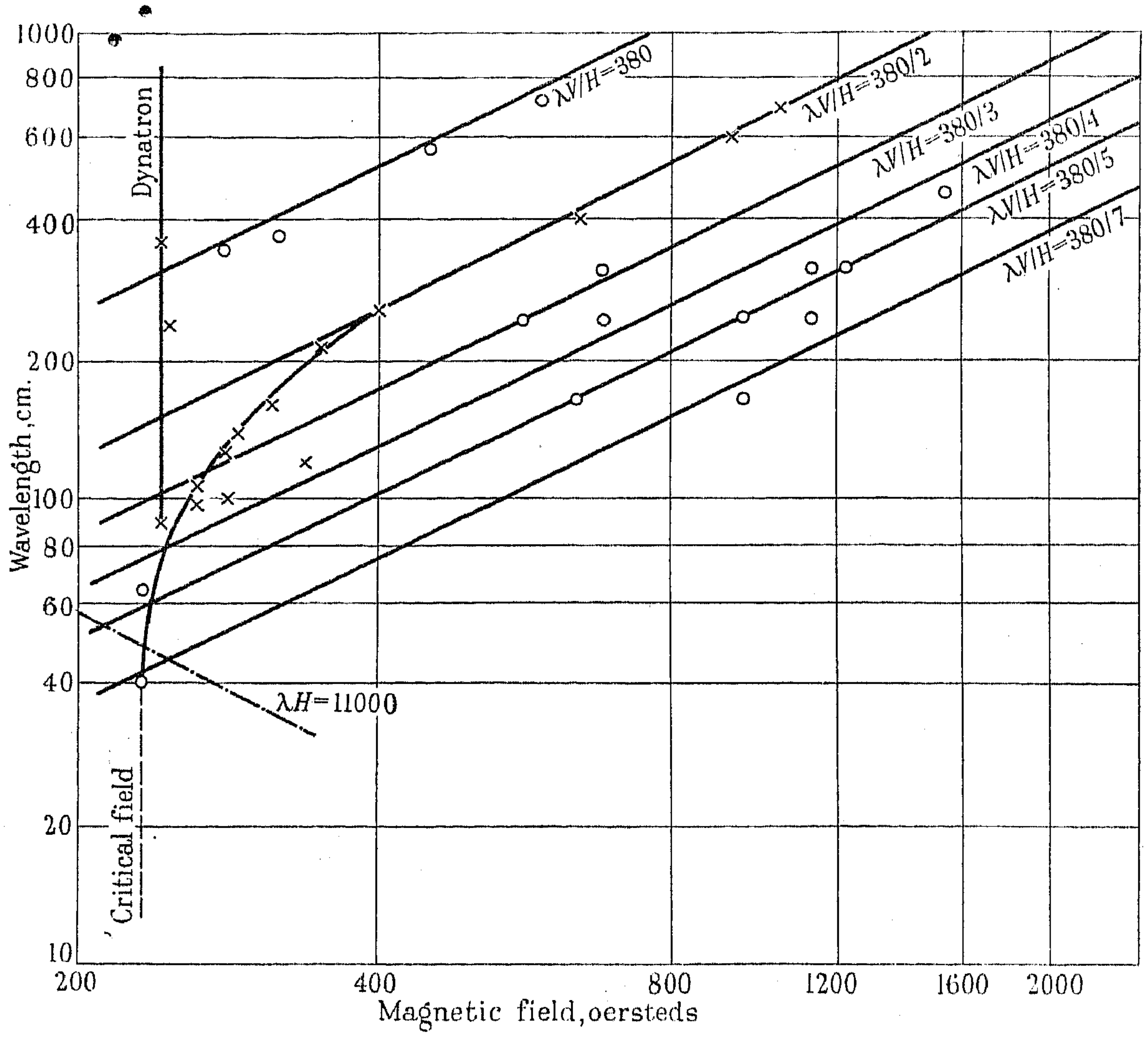

Fig. 7.—Wavelength of oscillations of 4-segment-anode magnetron.

V_a = 300 volts. Anode diameter 1 cm. (nominal).
× × Strong oscillations.
○ ○ Weak oscillations.

2-segment valve, the value of $\lambda V/H$ decreasing gradually as the wavelength of the oscillations decreases. The maximum value of $\lambda V/H$ is about 200 instead of 400 as in the case of the 2-segment valve of similar dimensions, thus corroborating relation (3).

The shortest wavelengths plotted in Fig. 6 are of the electronic type, and the same continuity between resonance and electronic oscillations as was found with the 2-segment valve is apparent. The electronic region was not investigated in as much detail as with the 2-segment valve, but a general similarity of behaviour was evident. A number of cases occurred in which the value of λH was about 7 000 and one wavelength persisted over a range of voltages, although in not so pronounced a manner as with the 2-segment valve.

Experiments were also made on a 4-segment valve,

(d) Relation between Wavelength of Resonance Oscillation and Anode Current

The experimental results given above show that the factor $\lambda V/H$ is a constant for large magnetic fields and decreases when for a given anode voltage the magnetic field and wavelengths are reduced. $\lambda V/H$ is a constant, therefore, over those parts of the anode-current/magnetic-field characteristic corresponding to small anode current, and the curved portions of the $\lambda V/H$ curves in Figs. 3 and 7 are confined to regions near the critical magnetic field for which the static anode current varies rapidly with change in magnetic field. It was considered that the variation in $\lambda V/H$ might be related directly to the static anode current. The static characteristic connecting the anode current i with the magnetic field H was measured for each experiment with a filament current sufficiently small to avoid oscillations. From the results obtained in this way it was possible to approximate to the static anode current appropriate to each wavelength of oscillation by determining the anode current corresponding to the magnetic field for maximum oscillation output. Typical results relating the wavelength to the static anode current are given on a logarithmic scale in Fig. 8. It will be noted that over a large range of wavelengths the experimental points for both 2- and 4-segment valves lie closely to the straight lines given by

$$\lambda^2 i = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

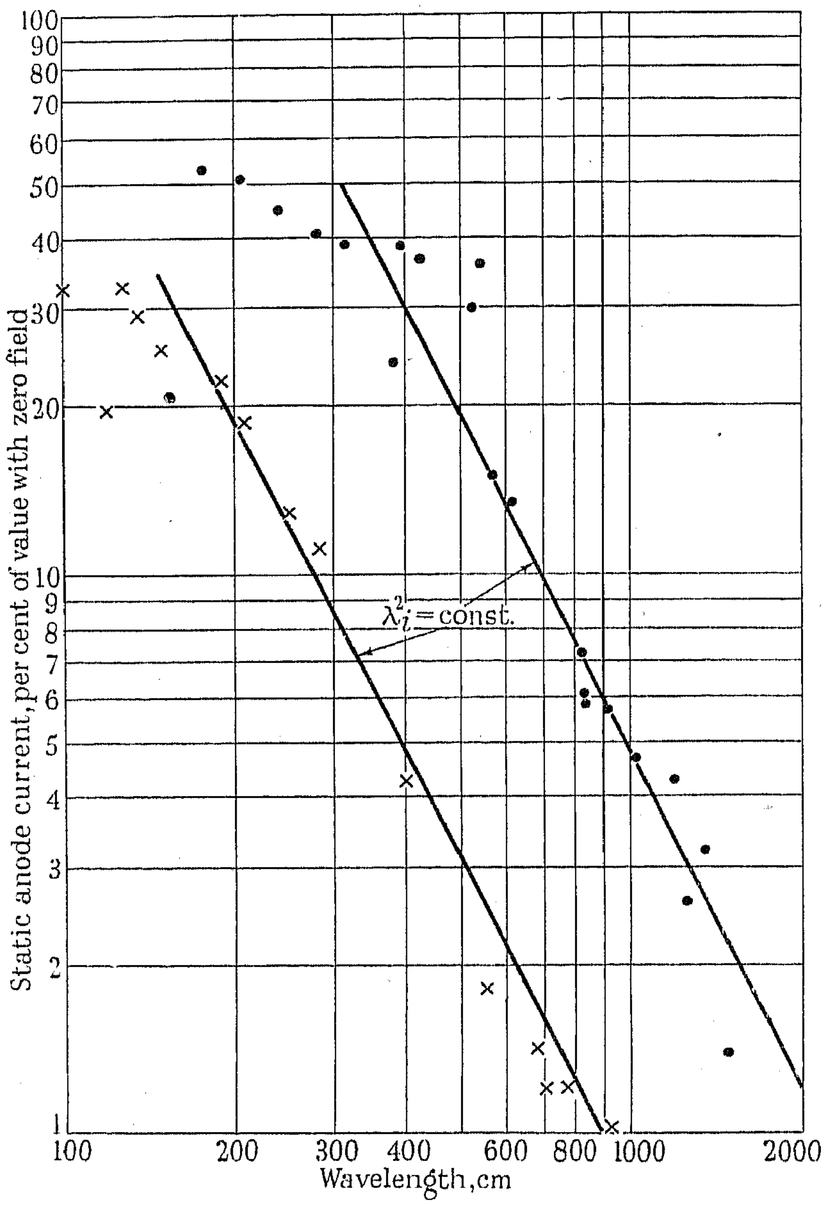

Fig. 8.—Static anode current plotted against wavelength for 2- and 4-segment-anode magnetrons.

● ● 2-segment. × × 4-segment.

The experimental results which do not agree with this relation correspond to points on the anode-current/magnetic-field characteristic where the anode current varies too rapidly with magnetic field intensity for the appropriate anode current for each magnetic field intensity to be accurately determinable. It is interesting to note that Gill[10] obtained an exactly similar relation to equation (6) above, between the wavelength of electronic oscillations in a positive-grid triode and the grid current. This additional similarity between electronic oscillations in triodes and resonance oscillations in magnetrons strengthens the hypothesis that the two types of oscillation are essentially the same.

Linder[11] has recently published theoretical formulae, supported by experimental results, which differ considerably from equation (6). The experimental results obtained by the authors, however, did not agree with Linder's theoretical formulae nearly so closely as with the purely empirical relation given above.

(e) Effect of Tilt

Most workers on the magnetron are agreed that the angle between the magnetic lines of force and the electrode axis is of importance for optimum conditions of oscillation and is not in general zero.[3, 7] There is not the same agreement, however, as to the circumstances under which this tilt becomes advantageous or even essential. The experiments described above were conducted with angles of tilt ranging up to about 7°. The effect of tilt was not, in general, simple, and did not always repeat as between two valves nominally identical and working under the same conditions. The main effect of tilting the valve seemed to be a broadening of the range of magnetic field over which oscillation took place, possibly because the static anode-current/magnetic-field characteristic curve slopes less steeply when the valve axis makes a small angle with the direction of the magnetic lines of force. Thus an oscillation which required a setting of magnetic field current accurate to ± 5 % when the valve had zero tilt might be maintained with a range of magnetic field current of, say, ± 20 % when the valve had a small tilt.

(3) DISCUSSION OF RESULTS AND CONCLUSIONS

The experimental analysis of the behaviour of the magnetron as a short-wave oscillator is extremely complicated owing to the large number of factors controlling the amplitude of the oscillations and also their wavelength; the last-named, in some cases, is determined by a combination of the anode voltage, magnetic field, the angle of tilt which the valve axis makes with the direction of the magnetic field, and the external circuit. One valve may show a marked tendency to oscillate at some particular frequency determined, by, say, the length of lead from anode to seal, which has apparently little effect in a second nominally identical valve. Even if only one type of oscillation was produced by the magnetron a complete tabulation of its properties would be a formidable task as the optimum condition for each oscillation depends on the appropriate adjustment of all the factors controlling the oscillations, and they can never be determined with any great accuracy. As the magnetron can be used to produce oscillations of two or three different types, sometimes simultaneously, an entirely satisfactory explanation of its behaviour determined empirically may be impossible.

The authors have not attempted to produce any hypothesis for the production of oscillations in the magnetron, but have confined themselves intentionally to an analysis of the results obtained in a large number of experiments. The following facts can be derived from this analysis.

(*a*) A full-anode magnetron can oscillate at a number of wavelengths with magnetic fields considerably greater than the critical value. The wavelength of these oscillations is almost independent of the external circuit, and the quantity $\lambda V/H$ may have any value given by K, $K/2, K/3 \ldots K/n$. For a valve having an anode diameter of 1 cm., K will have a value of about 400. Sometimes oscillations at two distinct wavelengths occur for the same magnetic field. These oscillations must all be regarded as of the so-called resonance type.

(*b*) Weak oscillations similar to those for the full-anode valve are found in multi-segment magnetrons. Presumably the valve acts as a single-segment magnetron under these conditions.

(*c*) With multi-segment valves the resonance oscillations of largest amplitude are those for which $\lambda V/H$ is approximately $1/p$ times the maximum value of $\lambda V/H$ for a similar-dimensioned valve having a full anode, where p is the number of pairs of anode segments, e.g. for a 1 cm. diameter valve $\lambda V/H$ for maximum oscillation amplitude is 400 and 200 for 2- and 4-anode-segment valves, respectively.

(*d*) The value of $\lambda V/H$ for the main resonance oscillations on a multi-segment valve decreases from a constant value at the longer wavelengths to a considerably lower value at the shortest wavelengths possible with this type of oscillation.

(*e*) Electronic oscillations obeying the law $\lambda H = 11\,000$ may occur at magnetic field intensities both greater and less than the critical field.

(*f*) The value of $\lambda V/H$ for electronic oscillations increases with the wavelength of oscillation, becoming the same as that for resonance oscillation at a common wavelength.

(*g*) A continuous variation of wavelength of electronic oscillations with voltage does not occur.

(*h*) The product of the square of the wavelength of resonance oscillations and the static anode current is constant over a considerable range of wavelengths.

(*j*) The investigation shows that the electronic and resonance types of oscillation are essentially the same or that the electronic oscillation is only a special case of resonance oscillation.

The ability of the multi-segment magnetron to act simultaneously as a single-segment-anode valve and, therefore, to give rise to a series of wavelengths of oscillation for one setting of magnetic field may account for the experimental observation that the output from a magnetron, although periodic, is seldom in the form of a sine wave.

Gill and Britton[1] have put forward the hypothesis that the resonance oscillations found in multi-segment magnetrons are due to a precession of the electrons within the valve set up by a combination of the static electric field between the filament and anode, the magnetic field, and the alternating potential between the anode segments due to the oscillations. The results described in the paper show that the same resonance oscillations occur both in single- and multi-segment anode valves having similar dimensions, but that by dividing the anode into a number of segments certain types of these resonance oscillations become predominant. As these resonance oscillations occur in a full-anode magnetron, it would appear that the precession of electrons occurs even in the absence of any alternating component of electric field between the segments of the magnetron, but that the effect becomes more pronounced on division of the anode into a number of separate segments.

Acknowledgments

The work described in this paper was carried out as part of the programme of the Radio Research Board, and thanks are due to the Department of Scientific and Industrial Research for granting permission for publication. The authors are also indebted to Mr. F. M. Colebrook, B.Sc., for advice on the method of presentation of the paper, and to Mr. C. G. Carter, B.Sc., for assistance with the measurements.

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A MAGNETRON OSCILLATOR WITH A COMPOUND FIELD WINDING*

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(*Paper first received 8th May, and in revised form 3rd November,* 1939.)

SUMMARY

The possibility of using the anode current to excite additional coils on the electromagnet of a magnetron oscillator and thus control the magnetic field is discussed, and a description given of experiments made with such an arrangement. The direct connection, in which the anode current assists the main magnetizing current, is shown to be of doubtful value; but the differential connection, in which the anode current opposes the main magnetizing current, when used in conjunction with a series resistance in the anode circuit, is shown to possess advantages as a generator of resonance oscillations. The number of turns on the coils and the magnitude of the series resistance can be so adjusted that the quotient of anode voltage to field intensity is approximately constant over the range of oscillation, which improves the operational stability and is likely to improve the frequency stability. The danger of the anode becoming overheated if a sudden reduction of the load on the valve takes place is also removed.

(1) INTRODUCTION

The magnetic field of a magnetron oscillator is commonly provided either by a permanent magnet or by an electromagnet energized by a direct-current supply independent of those of the anode or filament of the valve. In the former case the desired oscillation conditions are obtained by adjusting the anode voltage, while in the latter case either the anode voltage or the magnetic field may be adjusted. The use of an electromagnet consequently increases the flexibility of an oscillator at the expense of a decreased overall efficiency, and is generally preferable if the oscillator is to be used at a number of frequencies, although adjustable " permanent " magnets are sometimes used.

When an electromagnet is used the possibility arises of using either the filament or anode currents of the valve to provide all or part of the magnetic field necessary for the operation of the oscillator. The work to be described was undertaken to determine whether the anode current can usefully be employed partially to control the magnetic field when the valve is oscillating in the resonance condition, with which wavelengths of 40 cm. and upwards are obtainable. For the resonance type of oscillation the condition $\lambda V/H$ = a constant† must be satisfied, so that the prospects of improving the performance by this means seemed worth while investigating.

Oscillations in an electromagnet type of magnetron are generally obtained by adjusting the magnetizing current until the magnetic field reaches the required optimum, when the amplitudes of oscillation and anode current are maxima. The way in which the anode current supplied to the valve varies with the current in the coils of the electromagnet gives, therefore, a useful indication of the behaviour of the magnetron. A discussion of this, in the region of oscillation, is consequently given below for the cases of magnetrons with simple, direct compound, and differentially compound electromagnets, before going on to describe the experimental results obtained.

(2) RELATIONSHIP BETWEEN MAGNETIZING CURRENT AND ANODE CURRENT WITH RESONANCE OSCILLATIONS

(a) Simple Electromagnet

The relationship which exists between the exciting current of the electromagnet and the anode current of a magnetron valve oscillating in the resonance condition is shown diagrammatically in Curve 1 of Fig. 1. As the magnetic field is increased the valve begins to oscillate weakly at a condition corresponding to the point A and the anode current commences to rise; as the field is further increased an optimum is reached at B, where the anode current and amplitude of oscillation are at maxima, after which a still further increase of the field causes the oscillation to weaken, with diminishing anode current until the point C is reached, when oscillation ceases. When the field is decreased from a value greater than C the cycle of events is similar in the reverse direction.

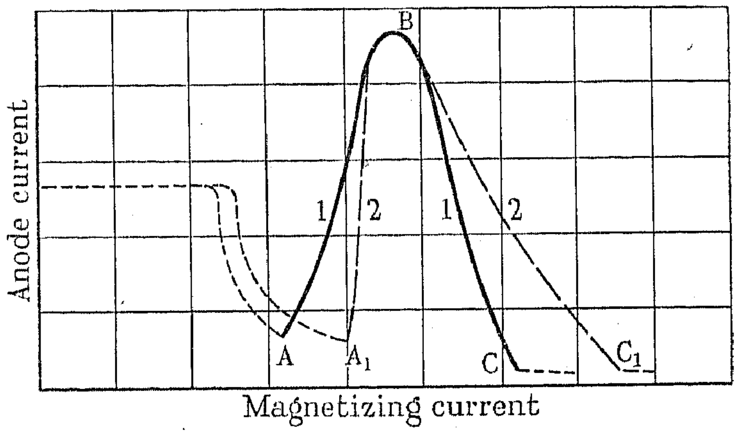

Fig. 1.—Relation between exciting current of simple electromagnet and anode current of magnetron.

If a magnetron valve is used under conditions of maximum output a sudden decrease of load is liable to cause an increase in the oscillatory voltage with excessive anode heating; or secondary emission due to filament bombardment may cause the valve to become unstable, with disastrous results. A series resistance is, therefore,

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[*] Official communication from the National Physical Laboratory.
Reprinted from *Journal I.E.E.*, 1940, **86**, p. 293.

[†] Where λ = wavelength of oscillation, V = anode voltage, H = magnetic field.

frequently inserted in the anode circuit; this prevents the current from reaching a dangerous value by absorbing part of the supply voltage. Curve 2 of Fig. 1 shows the effect of such a resistance on the action of the valve, on the assumption that the supply voltage has been increased to bring the anode voltage to the same value as for Curve 1 when the optimum oscillation condition is established. The value to which the field current must be brought before oscillations begin is increased from A to A_1, since the anode voltage with small anode currents is greater than before, but the curve rises more steeply to its maximum value, which again occurs at B. The decay of oscillations as the magnetizing current is further increased is delayed, owing to the rise in anode voltage as the anode current falls, until oscillations finally cease at C_1.

(b) Direct compound electromagnet

When the anode current of the valve is passed through coils on the electromagnet in the direction which increases the field due to the current in the main magnetizing coils,* the effect is similar to that of series resistance in the anode circuit. In the one case the anode current increases the value of H, and in the other it decreases the value of V, so that the effect on the quotient V/H is the same in both cases.

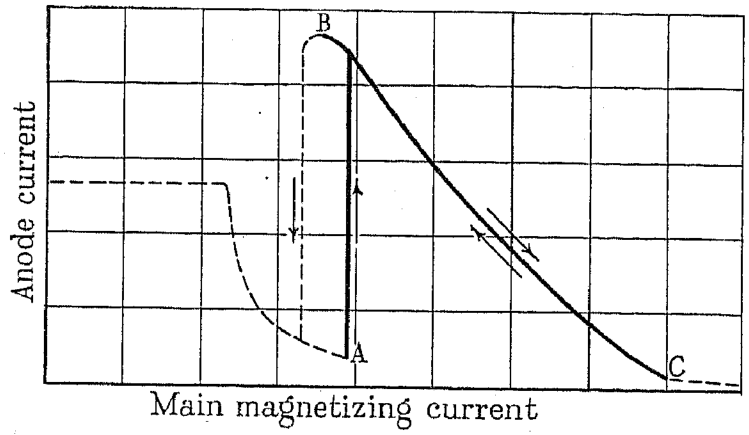

Fig. 2.—Relation between anode and main magnetizing currents of direct compound electromagnet.

An interesting possibility occurs if the anode current is passed both through coils on the magnet and through a series resistance. As explained in the previous Section, the effect of the series resistance is to increase the value of main magnetizing current at which oscillation begins, and this will outweigh the tendency of the additional coils to increase the magnetic field, since immediately before oscillation the anode current is relatively small. Consequently, when oscillation begins the rapid rise in anode current may increase the magnetic field to a greater value than that suitable for maximum oscillation, which would then be attainable by making a reduction in the main field current. A decrease of the main field current below the value necessary to bring the anode current to its maximum value must then cause an abrupt cessation of oscillation, since any decrease in anode current produces a reduction in magnetic field which in itself further decreases the anode current. The connection between anode and main magnetizing currents is shown diagrammatically in Fig. 2.

(c) Differential compound electromagnet

In addition to the possibility of using the anode current to energize field coils on the electromagnet and thus provide part of the required magnetic field, the anode current may be passed through the coils in the opposite direction, reducing the total magnetic field to a lower value than that obtained with the normal magnetizing winding only. Since a series resistance in the anode circuit causes the anode voltage to be a minimum when the current is a maximum, and differential connection of the field coils carrying the anode current will cause the total magnetic field to be a minimum when this current is a maximum (the main magnetizing current remaining unaltered), suitable proportioning of the series resistance and the number of turns on the field coils will cause the quotient V/H to remain constant over the whole range of anode current within which oscillation occurs. Resonance oscillations require a constant quotient of V/H, so that this arrangement appears likely to have advantages in securing both stability of operation and greater constancy of frequency of the oscillations generated.

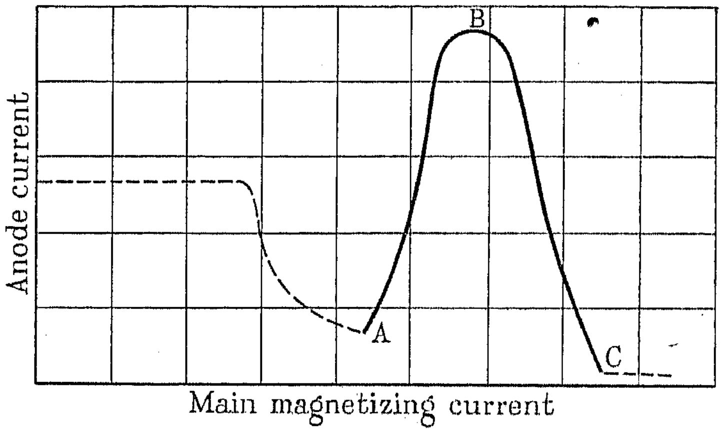

Fig. 3.—Relation between anode and main magnetizing currents of differential compound electromagnet.

Since the effect on the behaviour of the valve of the magnetizing coils in the anode circuit tends to be the reverse of that of series resistance, the probable curve between anode current and main magnetizing current, shown diagrammatically in Fig. 3, will be not dissimilar from that for the simple electromagnet magnetron, with the values of main magnetizing current increased.

(3) EXPERIMENTAL RESULTS

Experiments were made on the possible arrangements of magnetic field system described above, using 4 valves, each having a 4-segment anode, to give resonance oscillations of 1 metre wavelength. The general arrangement of field and anode circuits was as shown in Fig. 4. The main field coils on the limbs of the magnet consisted of about 3 000 turns of No. 16 S.W.G. copper wire, and additional coils of 20 000 turns of No. 28 S.W.G., through which the anode current passed, were placed on the poles. The normal conditions of operation required a field current of 3 amperes when the anode voltage and current were about 2 000 volts and 150 milliamperes respectively. Consequently, with 5 000 ohms in series with the anode and the current flowing in the additional coils in opposition to the main current, it was necessary to increase the supply voltage to about 2 700 volts and the main field current to 4 amperes. The percentage reduction in magnetic field due to the anode current coils was, therefore, approximately equal to the percentage reduction in anode voltage due to the drop in the series resistance. The oscillatory circuit consisted of a Lecher wire system bridged at a point about 60 cm. from the anode seals of the valve. With this circuit oscillations of about 40 cm. and 3·5 metres, as well as 1 metre, could be obtained by suitable adjustment of magnetic field. The load used was a de-capped 100-watt carbon-filament lamp which could be applied at any point on the circuit.

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* The use of this connection of the magnetizing winding to improve the modulation of the magnetron has been previously described—Patent No. 421652, Messrs. Marconi, Ltd., and A. W. Ladner, 1934.

Experiments were made on each valve, at 1 metre wavelength, with the field system used first as a simple electromagnet, the coils AA being disconnected, and then with the anode current flowing through these coils both in the direction to assist and to oppose the main magnetizing current in the coils MM.

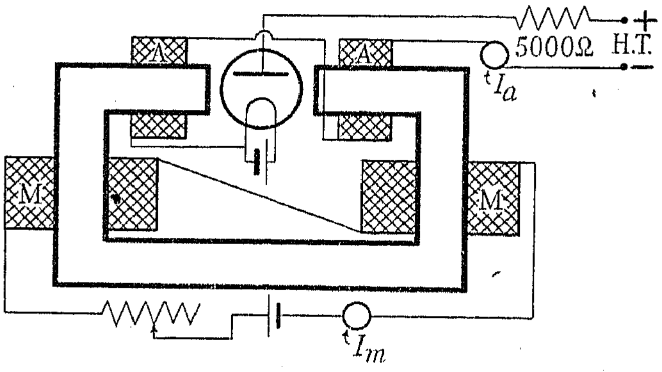

Fig. 4.—Compound magnetron.

(a) Simple electromagnet

The behaviour of the oscillator was quite normal, as outlined in Section 2(*a*), Fig. 1, curve 2. Oscillation commenced weakly, built up to a rather blunt maximum, and decreased relatively slowly as the magnetizing current was increased through the range over which oscillation occurred. The magnetizing current, H.T. supply, anode current and power output in the lamp load were noted with the magnetic field adjusted for maximum output. It was found generally that removing the load left the anode current unchanged, or even slightly increased, with a marked increase in anode heating. This effect would occasionally occur, apparently owing to some chance variation, without any actual movement of the lamp serving as a load; that is, the lamp would go black and the valve anode brighten without any considerable change in anode current such as would occur if oscillation ceased. Sometimes, however, oscillation ceased completely with removal of the load, and in such cases could only be started again by readjusting the magnetic field. The stability of operation appeared to be quite good, but no run of greater than half-hour duration was attempted, since it was inadvisable to leave the set unattended owing to the danger of increased anode heating with a chance decrease in load.

(b) Direct compound electromagnet

The previous experiments were repeated with the anode current flowing through the additional coils in the direction to assist the main field. It was found that oscillations could be obtained more readily if the main magnetizing current were reduced from too high a value than if it were increased from too low a value. In the latter case the behaviour was as outlined in Section 2(*b*), and a decrease of main magnetizing current from the value at which oscillation commenced caused an increase in output. The output increased as the magnetizing current was reduced, and then oscillation ceased abruptly at about the same output as previously obtained under similar conditions with the simple electromagnet, when the efficiency and the value of $\lambda V/H$ were also substantially the same as before. The effect of removing the load was almost invariably to cause oscillations to cease, which made it difficult to adjust the load to the optimum position on the circuit. The stability, as judged over relatively short periods, was good, provided that the field was adjusted to above the optimum value so that the output was appreciably below the maximum. Such adjustment did not involve any considerable loss in valve efficiency. When oscillating under these conditions a small decrease in anode current will decrease the magnetic field, and thereby tend to increase the anode current, and vice versa. Thus good stability is to be expected under these conditions. There was a tendency with some valves for a second peak of output to occur in this region with about two-thirds of the maximum output. It was not very pronounced, and when it occurred the combined magnetic field due to the coils A and M was almost the same for the two maxima.

(c) Differential compound electromagnet

The previous experiments were repeated with the anode current flowing through the additional coils in the direction opposing the main field. Oscillation started rather abruptly at almost maximum output when the main magnetizing current was increased to the necessary value, and disappeared almost equally abruptly when the current was further increased to a value about 10 per cent greater than that at which oscillation had begun. When the main magnetizing current was on the low side of the optimum value there was a marked tendency to instability of anode current, both when oscillations were just starting and just on the point of ceasing. This tendency was much less marked when the main magnetizing current was greater than the optimum value.

When the load was removed the anode current dropped to about one-half of its previous value, rising again to its full value when the load was replaced. This effect occurred with each of the four valves used in the experiments and is of the greatest value. The output and efficiency seemed somewhat greater than were obtained when using a simple electromagnet, but this may have been due to the ease with which the load could be adjusted to its optimum position in the circuit, and to the tendency of the magnetron to adjust itself automatically to maximum output with this connection. The stability was very good. For the first hour or so after switching on an occasional adjustment of the resistance in the main magnetizing circuit was necessary to compensate for the temperature rise in the field coils and maintain the current constant, but when thermal equilibrium was reached the oscillator ran completely unattended for periods of an hour or more without any noticeable alteration in output. Experiments were not extended over a longer period than two hours, but there seems no reason to anticipate that, given constancy of supply voltages, the oscillator would not continue to run indefinitely.

The results obtained on one of the four valves used in the experiments described above are given in the Table. It was a 4-segment valve with an anode diameter of 1 cm. and length of 2·5 cm.; the supply voltage was 2 500, and 5 000 ohms were inserted in series with the anode. The wavelength was 1 metre.

(4) DISCUSSION OF RESULTS, AND CONCLUSIONS

A magnetron oscillator with a compound field winding, connected so that the anode current increases the magnetic field, possesses the disadvantages that its maximum output occurs under conditions such that either a slight decrease in magnetic field or removal of the load will, in most cases, bring about a cessation of oscillation. This tendency to stop oscillating with removal of the load is at the same time an advantage, since the valve is thus completely protected against the danger of overheating which frequently occurs with a simple magnetron under these circumstances. Thus a direct compound connection may be of use as a safety device serving substantially the same purpose as a series resistance in the anode circuit, without requiring so great an increase in anode supply voltage and with a greater overall efficiency.

Table

Connection	Magnetic field, oersteds			Anode		Output, watts	Efficiency, %	$\frac{\lambda/V}{H}$
	Main coils	Anode coils	Total	Current (mA)	Volts			
Simple	1 000	0	1 000	150	1 750	70	27	175
Direct compound	680	320	1 000	120	1 900	60	26	190
Differential compound ..	1 270	380	890	140	1 800	70	28	200

Note.—The four valves used showed considerable differences in efficiency, ranging from 25 % to 60 %, due presumably to differences in construction.

A differential connection, in which the anode current opposes the main current, used in conjunction with a series resistance in the anode circuit possesses important advantages. If the number of turns on the coils and the magnitude of the series resistance are so chosen that a change in anode current produces an equal proportional change in both magnetic field and anode voltage, the quotient V/H remains substantially constant over the whole range of oscillation, and output remains almost constant at its maximum value over this range. Removal of the load usually leads to a drop in anode current without interruption of the oscillations; there is thus complete protection against the danger of the valve overheating in these circumstances while the full oscillations are resumed when the load is restored.

There remain to be considered two disadvantages not directly connected with the oscillator as such. The resistance of the compound field coils must be appreciable (in the magnet system described it was 1 000 ohms), and consequently there is a difference of potential between the negative H.T. supply and the filament of the valve. There is also an increased power consumption, both in the anode circuit and main field circuit, which was nearly 200 watts in the experiments described. The drop in overall efficiency thus caused is not, however, of very great importance since the efficiency of the valve itself is not affected. The possible anode dissipation for a given valve is fixed; the valve efficiency determines the maximum output available, and is of the utmost importance. An additional loss, even of a few hundred watts, in the other parts of the magnetron equipment is not serious if corresponding advantages are obtained.

It may safely be said, therefore, that a magnetron having a differential compound electromagnet of the type described has advantages over the simple magnetron which considerably outweigh the disadvantages, and such a magnetron should be of use as a generator of resonance oscillations of short wavelength.

Acknowledgments

The work described in this paper has been carried out as part of the programme of the Radio Research Board, and the paper is published by permission of the Department of Scientific and Industrial Research. The author is indebted to Dr. R. L. Smith-Rose and Dr. J. S. McPetrie for their advice during the investigation.

THE IMPEDANCE OF THE MAGNETRON IN DIFFERENT REGIONS OF THE FREQUENCY SPECTRUM[*]

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(*Paper first received* 17*th July, and in revised form* 20*th October*, 1939.)

SUMMARY

A description is given of a series of measurements on the impedance of the magnetron and its relation to the operating conditions in different regions of the frequency spectrum. The object of the experiments was to determine the direct properties of the valve alone, in order to understand and explain the mechanism of operation of the magnetron. This has not been completely possible, and it appears that a full explanation is difficult unless existing theories of electron motion in the magnetron, especially under dynamic conditions, are much extended.

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(1) INTRODUCTION

The static characteristic of the split-anode magnetron is such that the valve behaves as a negative resistance to potential differences between the anode segments, when these are at a mean potential V, and this property persists in a range of H (magnetic field strength) from about the cut-off[†] value up to indefinitely strong fields. The negative resistance in this region of so-called dynatron oscillation is sensibly independent of the wavelength provided this is long enough, and it persists down to wavelengths of the order 1 m. These oscillations, while giving high efficiency, result in only medium power outputs at comparatively long wavelengths.

The magnetron also has negative-resistance properties which are different from those mentioned above, and these are distinguished by the fact that they are only partly dependent on external conditions. It exhibits these properties from wavelengths of the order of 25 cm. up to indefinitely long wavelengths, and this regime is referred to as that of resonance oscillations. Here the magnetron is capable of giving with high efficiency the largest obtainable power outputs at ultra-short wavelengths.

In addition, the magnetron has negative-resistance properties at wavelengths which are such that the periodic time is closely related to the electron transit time. The valve properties in this regime, usually known as that of electronic oscillations, are practically independent of external conditions and relate to wavelengths usually less than 25 cm. Although the efficiency and power output are low, these oscillations include the shortest continuous waves that can be generated.

Most of the experiments described in this paper were made on a Marconi-Osram CW10 magnetron. The anode had a length of 2 cm., a diameter of 1 cm., and the segment gaps were about 2 mm. wide: the valve was rated for a dissipation of 50 watts with a maximum anode voltage of 1 200 volts and a maximum filament emission of 80 mA. Since the anode was in two segments the valve formed a symmetrical impedance; and the experimental results, unless otherwise stated, refer to voltage amplitudes and impedances for the complete valve, i.e. from one anode segment to the other.

(2) IMPEDANCE IN THE DYNATRON REGIME

The impedance (which is resistive) and properties of the magnetron in the dynatron regime and their relation to the static characteristics have been determined by the author,[1] whose measurements at short wavelengths have shown that until the wavelength falls below about 100 m. the resistance is constant, and after that it rises. This result is not unlike that given by normal dynatron and regenerative valve oscillators, and is a consequence of the electron transit time becoming an appreciable fraction of the cycle. Determinations of the resistance for very high values of magnetic field give results shown by the lower dotted line in Fig. 1. Up to a limiting value any increase of amplitude results in a decrease of resistance, and for

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[*] Reprinted from *Journal I.E.E.*, 1940, **86**, p. 297.

[†] If e and m are respectively the charge and mass of the travelling particles and c is the velocity of light *in vacuo*, then for cylindrical electrodes the cut-off value H_c of magnetic field is given by

$$V = \frac{eH^2a^2}{8mc^2}\left(1 - \frac{b^2}{a^2}\right)^2$$

which becomes $H_c^2/181$ in the case of electrons and when the anode radius a is 5 mm. and the cathode radius b is 0·065 mm.

small amplitudes the resistance is negative (and of very low value) close to the cut-off field. There does not appear to be a complete explanation for the existence of these properties, but it is probably due in some measure to the distortion of the electrostatic field by the potential difference between the segments, as has been suggested by E. W. B. Gill and K. G. Britton[2] and by H. Zuhrt.[3]

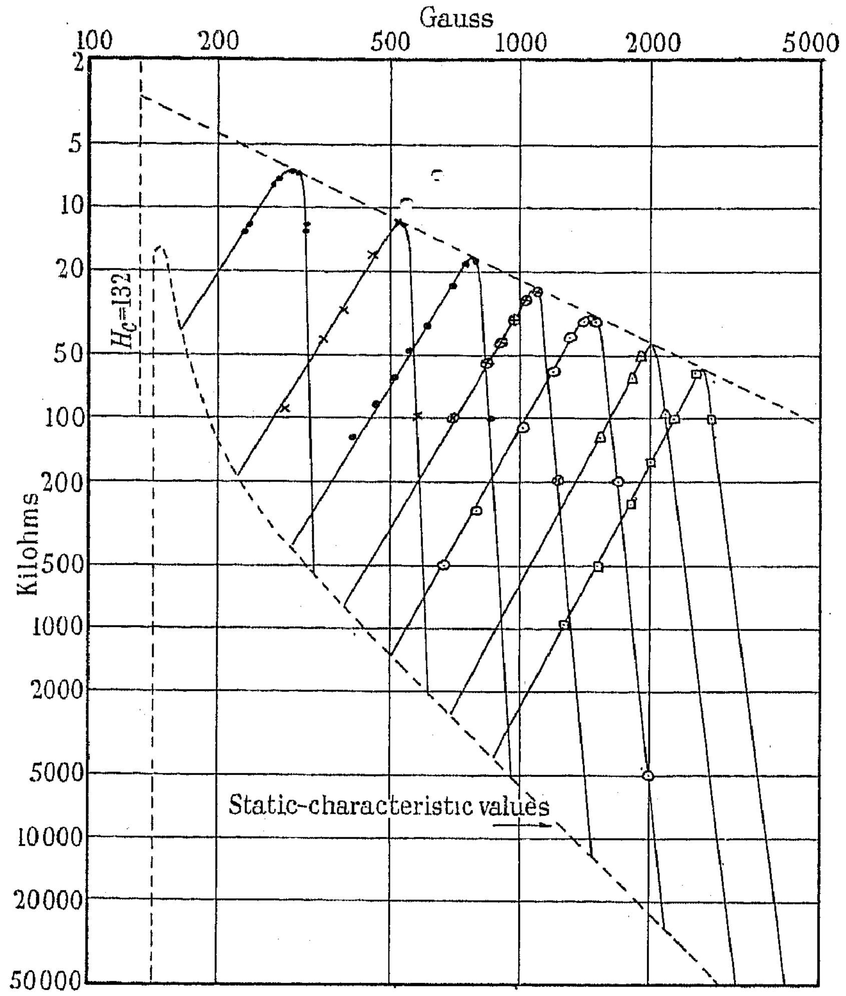

Fig. 1.—Variation of negative resistance with field strength at various different wavelengths.

Anode voltage, 100 volts.
Filament emission, 10 mA.
Amplitude, 29 volts (r.m.s.).
Wavelength 9·27 m. —●-●——●-●—
Wavelength 14·7 m. —×—×—×—
Wavelength 23·2 m. —●—●—
Wavelength 33·1 m. —⊕—⊕—
Wavelength 43·3 m. —⊙—⊙—
Wavelength 61·8 m. —△—△—
Wavelength 80·3 m. —⊡—⊡—

Since it will be useful later, it is instructive to consider, under static conditions, the radius of the electron cloud when $H >> H_c$. If the cloud radius is r and the emission is assumed small, so that

$$\frac{V_r}{V} = \frac{\log r/b}{\log a/b}$$

then

$$\frac{r}{a} = \frac{H_c}{H} \cdot \frac{[1 - (b^2/a^2)]}{[1 - (b^2/r^2)]} \sqrt{\left(\frac{\log r/b}{\log a/b}\right)}$$

and for the valve used we have the values shown in Table 1, the last line giving for reference the measured anode current when both segments were at 100 volts with a filament emission of 10 mA. Thus at the higher field values considered in Fig. 1 the electron orbits are not far removed from the cathode.

It can be shown that with a finite emission the radius of the cloud is less and that if the space charge is sufficient to cause the field at the cathode to become zero then the cloud radius tends to one-half the above values. The conditions obtaining at cut-off, and the effect of tilt, have been examined by the author[4] and it is expected that tilt (*a*) tends to reduce space charge by causing electrons to move longitudinally and finally leave the inter-electrode space, (*b*) inclines the axis of the cylindrical cloud and brings it nearer the anode to an extent depending on the angle of tilt and the length of the electrodes.

(3) IMPEDANCE IN THE RESONANCE REGIME

(a) Method of Measurement

This was carried out by connecting the anode segments to an external impedance consisting of a symmetrical double-rotor condenser (capacitance of each half variable between 30 and 200 $\mu\mu$F) and a coil (inductance between 0·9 and 18 μH) the centre point of which was kept at a steady potential. The dynamic resistance of this circuit, when connected to the valve with its filament cold, was determined as a function of wavelength. This was made by the distuning method or by connecting metallized resistors in parallel with the coil (and as close as possible to the valve terminals), or by inserting small series resistances (of eureka, No. 40 S.W.G.) in the lead connecting the two halves of the condenser. Suitable corrections were made for the inevitable distributed inductances and capacitances of the circuit, and the agreement of the three methods was then very good. The voltage amplitude was measured by a pair of small diodes which rectified the radio-frequency voltage, the resulting steady voltage across the load condenser being read by a high-resistance voltmeter. This instrument, which read sensibly peak voltage, was calibrated on a sinusoidal 50-cycle supply and the readings were assumed valid at the wavelengths employed. The voltmeter was connected across the valve terminals, and it will be noted that the damping resistance (about 5 MΩ) and input capacitance (about 1·8 $\mu\mu$F)

Table 1

H/H_c	1·0	1·8	4·0	7·0	12·8	27·2	36·3	58·8	∞
a/r	1	2	5	10	20	50	60	70	77
r/b	77	38·5	15·4	7·7	3·85	1·54	1·28	1·10	1·00
$I_a(\mu A)$	10^4	25	5·0	1·1	0·1	0·003	—	—	—

were distributed equally across the two halves of the coil.

Much care was devoted to a choice of a suitable method of measuring the absolute valve resistance. Since the magnetron in this regime is equivalent to a complex non-linear impedance, the distuning method (capacitance or frequency variation) or methods similar to that due to P. W. Willans could not be employed. Finally, two methods were used according as the valve resistance was less than or greater than the rejector resistance of the external circuit. In the first, the valve and circuit together acted as a generator whose output voltage at various wavelengths was maintained at any assigned and constant value by placing an appropriate resistor in parallel with the core; and since the circuit has already been calibrated its rejector resistance is known at any wavelength when shunted by the known resistor. As the circuit power-factor was not large the fluctuations of anode potential are very nearly simple harmonic, and thus the fundamental component of negative resistance of the magnetron at any wavelength and amplitude is equal to the known rejector resistance. In the second method a voltage of the desired magnitude was induced from a separate electrostatically screened generator, self-oscillation of the magnetron being prevented by inserting sufficient resistance in the circuit. Then the filament of the magnetron was lit and the voltage amplitude brought back to its initial value by placing a suitable resistor in parallel with the coil, the circuit being retuned by slight adjustment of the condenser. The resistance necessary to do this is equal to the negative parallel resistance of the magnetron.

To reduce the uncertainties of measurement and the confusion which arises due to the different regimes in the magnetron associated with very short wavelengths, most measurements were made at comparatively long wavelengths. This necessitated the use of very strong magnetic fields, and values up to 3 500 gauss were provided by an iron-cored air-gap electromagnet of conventional form but of design appropriate to such intense fields.

(b) Wavelength and Anode Voltage

The negative resistance in the resonance regime is shown in Fig. 2 as a function of wavelength for three values of H. The value of E (r.m.s. voltage amplitude) chosen was such that the potential fluctuations of each anode were about $V/10$. It will be seen that the value of $-R$ (negative resistance) increases with λ (wavelength) in a comparatively gradual manner so long as λ is greater than a certain value depending on H, and that when λ is less than this value $-R$ increases much more rapidly; in either case $-R$ tends to the value given by the static characteristics, this being too high to be shown on the diagram. The minimum values of $-R$ lie on a line of unit slope (shown dotted), but it will be observed that for any given condition the value of λ is not critical and the magnetron would function well as a generator over a considerable range of wavelength if connected to a circuit of sufficiently high rejector resistance.

The existence of a sharply defined minimum wavelength is indicated from the theoretical analysis of the electron paths,[5] while some light on the mechanism of these oscillations has been given by E. W. B. Gill and K. G. Britton[2] and J. S. McPetrie.[6] It is generally accepted that resonance oscillations are found only when λ (in cm.), V, and H have values given by $\lambda V/H = K$, where K is a constant depending on the diameter and number of segments of the anode, but the results given show that negative resistance occurs over a considerable range of K. It will be convenient to use K as defined in discussing the experimental results, and thus for the conditions of Fig. 2 negative resistance occurs when K lies between 160 and 800. Measurements on the positions of minimum negative resistance give $K = 266$ when $E/V = 0{\cdot}45$, $K = 325$ when $E/V = 0{\cdot}136$ (as in Fig. 2), and $K = 405$ when $E/V = 0{\cdot}026$.

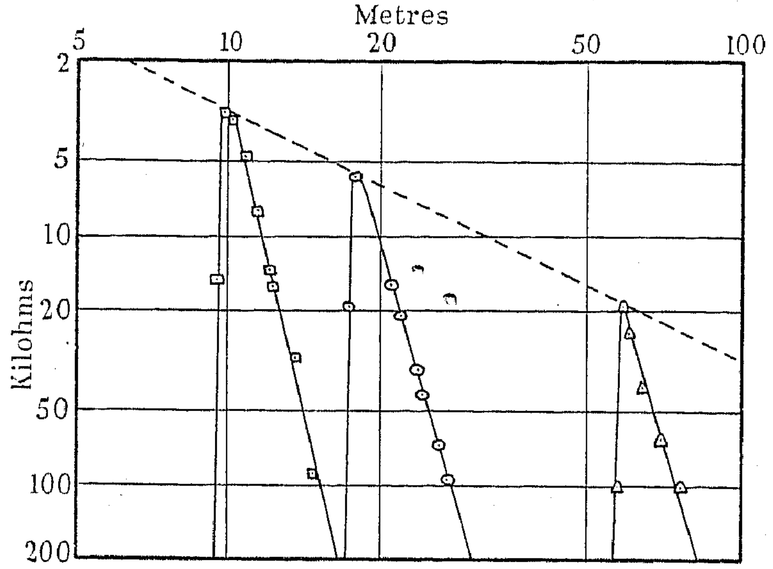

Fig. 2.—Variation of negative resistance with wavelength.

Anode voltage, 100 volts. Filament emission, 10 mA. Amplitude, 13·6 volts (r.m.s.).

Field strength 320 gauss —⊡——⊡—
Field strength 560 gauss —⊙——⊙—
Field strength 1 790 gauss —△——△—

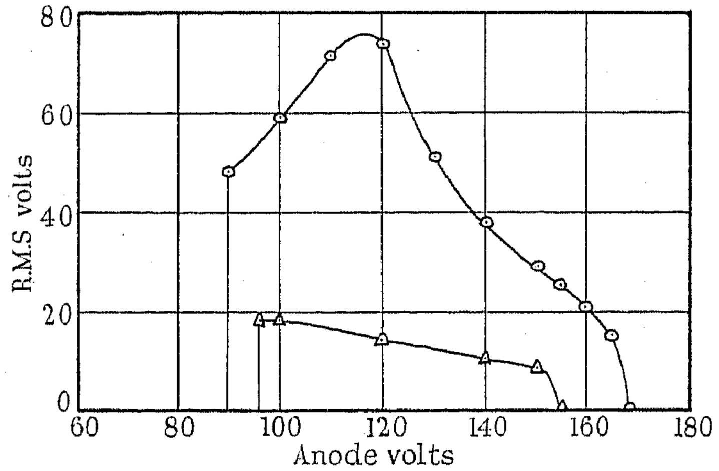

Fig. 3.—Variation of voltage amplitude with anode voltage.

Field strength, 2 215 gauss. Filament emission, 10 mA. Wavelength, 61·8 m

Negative resistance 115 kΩ —⊙——⊙—
Negative resistance 24·8 kΩ —△——△—

The above curves were repeated for other anode voltages between 50 and 200 volts, and the values of $-R$ obtained were almost identical with those given above provided E/V was kept the same and H was altered with V so that K remained the same. Similar curves can be obtained when V is varied (instead of λ), but it is useful in this case to consider a fixed resistance and examine the changes in amplitude. Results for two values of $-R$ are given in Fig. 3.

(c) Magnetic Field and Voltage Amplitude

These " resonance " curves can also be derived when H is varied as in Fig. 1 for seven different wavelengths, from which it will be seen that the minimum value of $-R$ is proportional to λ as indicated by the upper dotted line. For a given value of λ there is only a restricted range of H in which the valve exhibits marked negative resistance; nevertheless, it is likely that the proportionate increase of $-R$ is not attributable to the increase of λ *per se* but to the effect of going further along the curved foot of the cut-off curve with the subsequent contraction of the electron cloud. As this value of H is proportional to V, while H_c varies as $\sqrt{V}$, the operating point moves along the curved foot as V is made larger.

Since negative resistance is not found for $H < H_c$ the Figure shows that for any value of V there is a minimum wavelength. The positions of minimum resistance are found to be given by $K = \lambda V/H = 300$, and as $H_c = \sqrt{(181\ V)}$ the corresponding minimum wavelengths are given by $\lambda H_c = 54\,000$. Thus the minimum wavelength decreases as H_c (and hence V) increases. It will be seen later that a similar relation obtains for the wavelength of the electronic oscillations, but the value of the constant is then about 11 000. Provided sufficient field strength is available there appears to be no maximum wavelength to the resonance regime, and negative resistance (but of a very high value) would be found at indefinitely long (but not infinite) wavelengths. Accordingly the fact that self-oscillation is not encountered at long wavelengths is not due to any restriction by the properties of the magnetron; but is due to the lack of adequate circuits and the difficulty of providing the necessary intense fields.

Measurements show that the result of repeating the curves of Fig. 1 with other amplitudes is that $-R$ tends to increase with E, and since the valve impedance is given by the vector sum of the parallel impedances of the dynatron and resonance regimes the importance of carrying out these measurements at sufficiently long wavelengths to avoid confusion will be realized.

The curves in Fig. 4(*a*) show the relation between the fundamental component of negative resistance and the voltage amplitude, for three fields ranging from 11·3 to 16·8 H_c. It will be seen that the negative resistance is proportional to the amplitude so long as this is greater than a lower limit (which increases with field strength), and it should be understood that the almost vertical rise in negative resistance is a real effect. Since the relations result in straight lines directed to the origin it is evident that the fundamental component of anode current is constant whatever the voltage between the segments. It has recently been shown theoretically by G. Hara[7] that in this regime the magnetron tends to have constant-current properties.

On the other hand, the mean anode current, as in Fig. 4(*b*), increases with voltage amplitude in a linear manner, and it should be noticed that it may be much less than the fundamental component; thus when $H = 1\,950$ the latter is always 0·61 mA (r.m.s.), whereas the input current can have any value (within limits) greater than 0·13 mA. A wave-form consistent with these results is difficult to conceive, and it is likely that the fluctuating component of anode current is not caused by the conduction effects familiar in normal valves. It is observed that the valve resistance does not decrease indefinitely as the amplitude and field strength are reduced, but tends to a minimum. Fig. 5 shows that the relation between these minimum resistances and wavelength is linear and results in a value of K of 282.

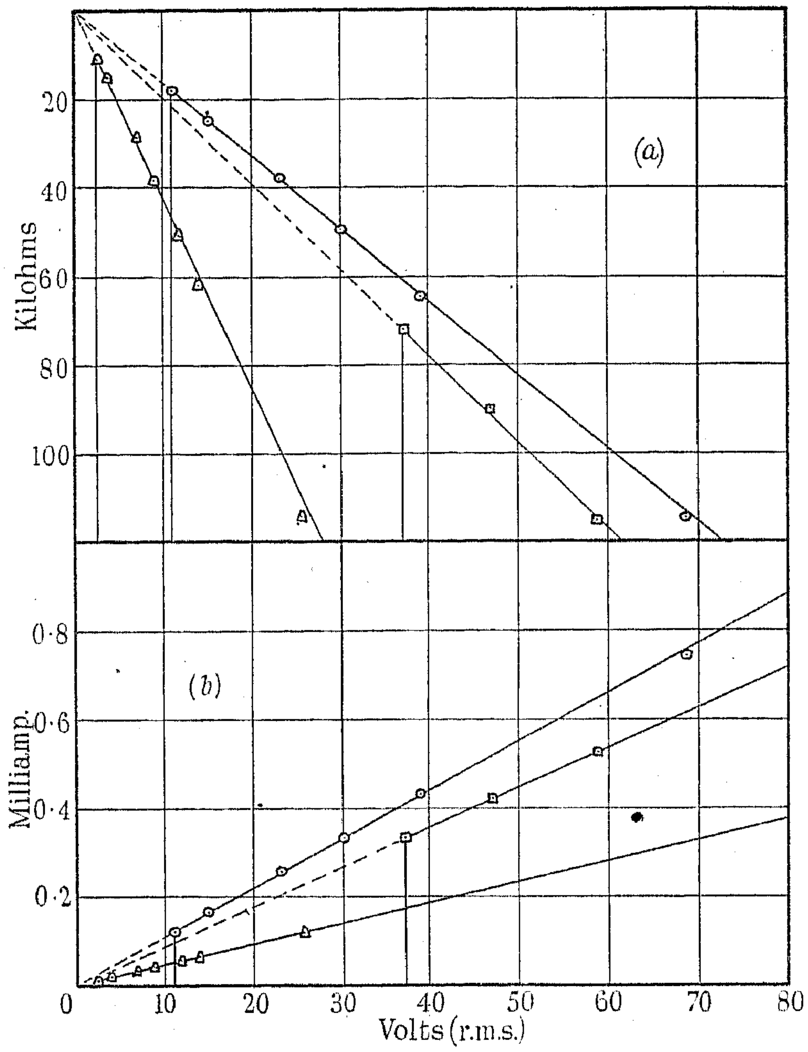

Fig. 4

(*a*) Variation of negative resistance with voltage amplitude.
(*b*) Variation of input current (d.c.) with voltage amplitude.

Anode voltage, 100 volts.	Field 1 525 gauss —△——△—
Filament emission, 10 mA.	Field 1 950 gauss —○——○—
Wavelength, 61·8 m.	Field 2 260 gauss —⊡——⊡—

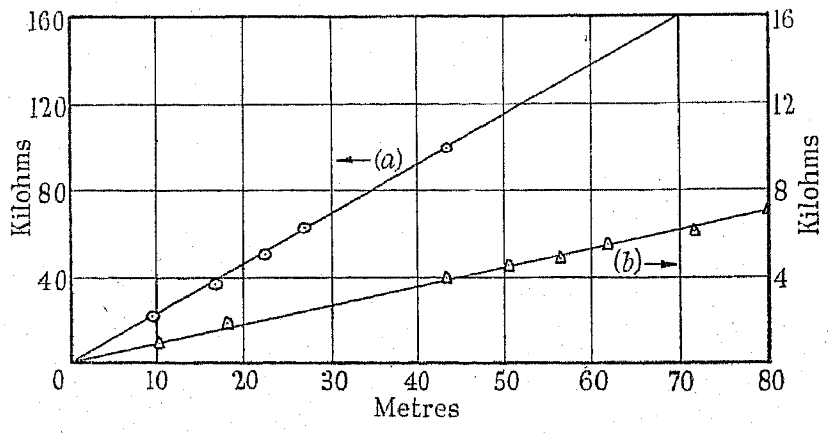

Fig. 5

(*a*) Variation of optimum load resistance with wavelength.	(*b*) Variation of minimum negative resistance with wavelength.
Anode voltage, 200 volts.	Anode voltage, 100 volts.
Field strength, optimum.	Field strength, optimum.
Filament emission, 10 mA.	Filament emission, 10 mA.

(d) Filament Emission and Angle of Tilt

The previous results have been obtained with I_e (total filament emission) large enough to make the valve resistance sensibly independent of it. Fig. 6(*a*) shows how $-R$

decreases, as I_e increases, to a blunt minimum and then slowly increases for large values of I_e. It is also found that λ varies with I_e, and by measuring the minimum value of λ (which is well defined, as Fig. 2 shows) for given conditions the results of Fig. 6(*b*) were obtained. It will be seen that the wavelength for a given amplitude and field increases as I_e decreases, which is the opposite to what would be expected if this "resonant" wavelength were directly related to the electron transit-time.

Fig. 7(*a*) shows how I_a (mean anode current) and the power E^2/R (the input to a load resistance R) vary with emission, the measurements being taken with a constant value of resistance. It will be seen that I_a and E^2/R reach maximum values when I_e is near the optimum value. A point of interest is that I_a is normally only slightly less than I_e, and thus the input current to the valve must be nearly constant over the cycle, and that its wave-form must be sensibly rectangular: it is probable that most of the current flows alternately to either anode segment during the half-cycle when it is at the lower potential. Attempts to verify this directly by observing the wave-form of the input feed current by an oscillograph or by determining the Fourier components (if any) by suitable tuned circuits have only been partly successful. When this is considered with the effect of voltage amplitude and the small radius of the electron cloud it seems that the electron motion induces a constant radio-frequency current in the anode circuit while the input current assumes a value directly related to the power E^2/R. It should be stated that the optimum values of I_e, particularly at the longer wavelengths, are only a very small fraction of the normal space-charge-limited current appropriate to the steady anode potentials, and thus there appears to be some condition in the valve which tends to increase the space charge.

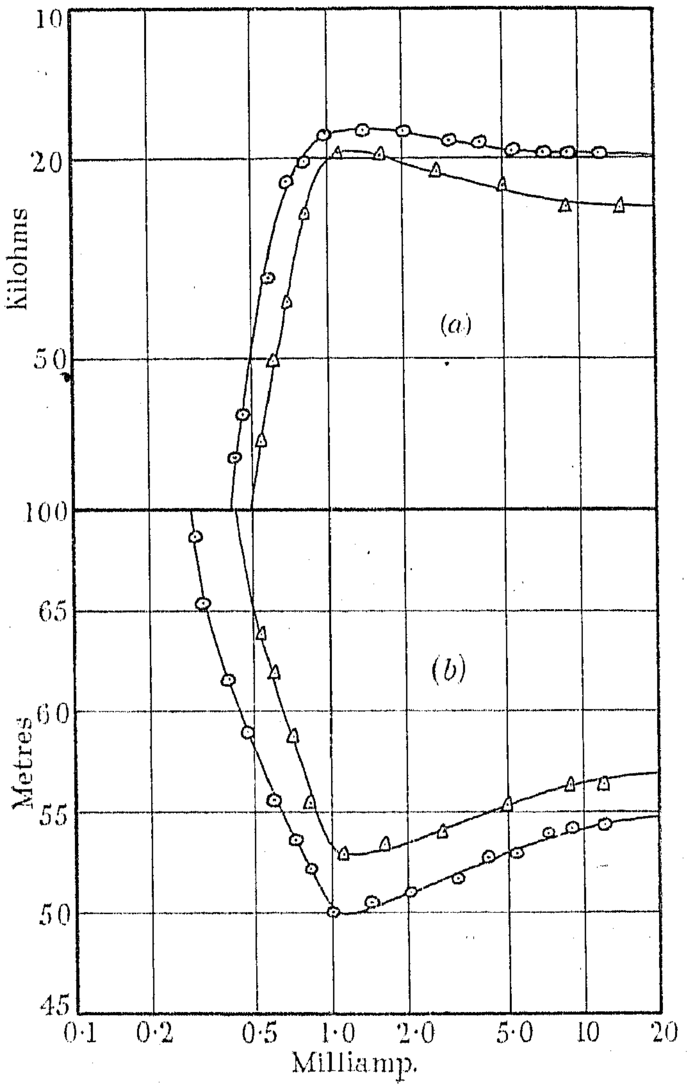

Fig. 6

(*a*) Variation of negative resistance with filament emission.
(*b*) Variation of wavelength (minimum) with filament emission.
Anode voltage, 100 volts. Field strength, 1 790 gauss. Amplitude, 13·6 volts (r.m.s.). Tilt zero —⊙—⊙—. Tilt 4° —△—△—

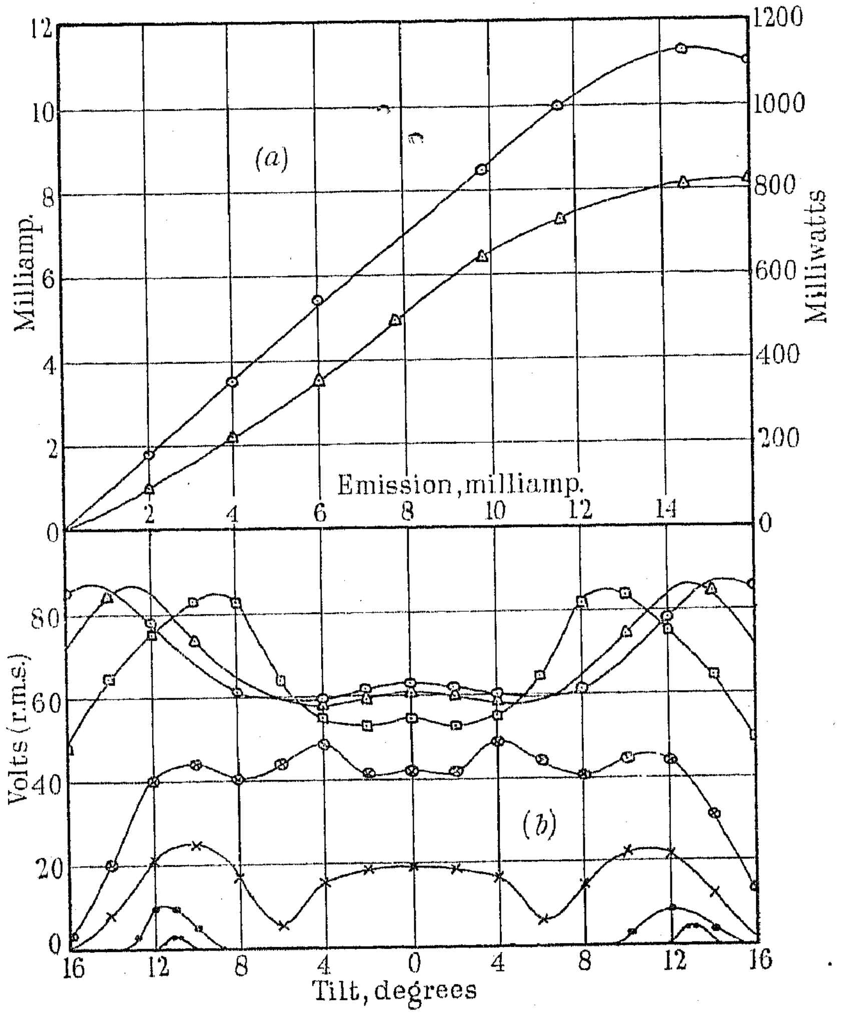

Fig. 7

(*a*) Variation of input current and of power with filament emission.
Anode voltage, 200 volts. Resistance, 18·6 kΩ.
Wavelength, 9·27 m. Tilt, zero.
Field strength, optimum. Input current —⊙—⊙—
Power, —△—△—

(*b*) Variation of amplitude with angle of tilt.
Anode voltage, 100 volts. Emission 5 mA —△—△—
Wavelength, 61·8 m. Emission 2 mA —⊡—⊡—
Field strength, optimum. Emission 1 mA —⊕—⊕—
Resistance, 115 kΩ. Emission 0·5 mA —×—×—
Emission 10 mA —⊙—⊙— Emission 0·2 mA —●—●—
Emission 0·1 mA —●-●—●-●—

It will have been seen that the changes due to increasing α (angle of tilt of field) are similar to those due to decreasing the emission, a result in agreement with one of the known effects of tilt on space charge. Small tilts are found not to cause appreciable changes in the valve properties (i.e. zero tilt is always a maximum or minimum point), but when α is large the effects are complex, as shown in Fig. 7(*b*), where the voltage amplitude, for a convenient value of resistance, was measured as a function of tilt. It will be seen that at zero tilt the amplitude rapidly decreases with emission when this is less than about 5 mA, and if I_e is less than about 0·25 mA there is no amplitude for which $R = -115$ kΩ. The humps near $\alpha = 12°$ show that tilt can decrease the negative resistance in certain cases, and this effect is related to the electron-cloud radius since when this is large (as when H and hence λ are small) tilt causes no improvement. This agrees with theory since, when the cloud radius is small, tilt causes the ends to approach more closely to the anode, without, however, causing the electrons to collide—as happens when the cloud is originally just within the anode sheath. It was further noticed that the optimum value of H decreases as α increases, and it is of interest to point out how apparent dissymmetry in the valve is much reduced as the emission increases—a fact not unique to the results of Fig. 7(*b*).

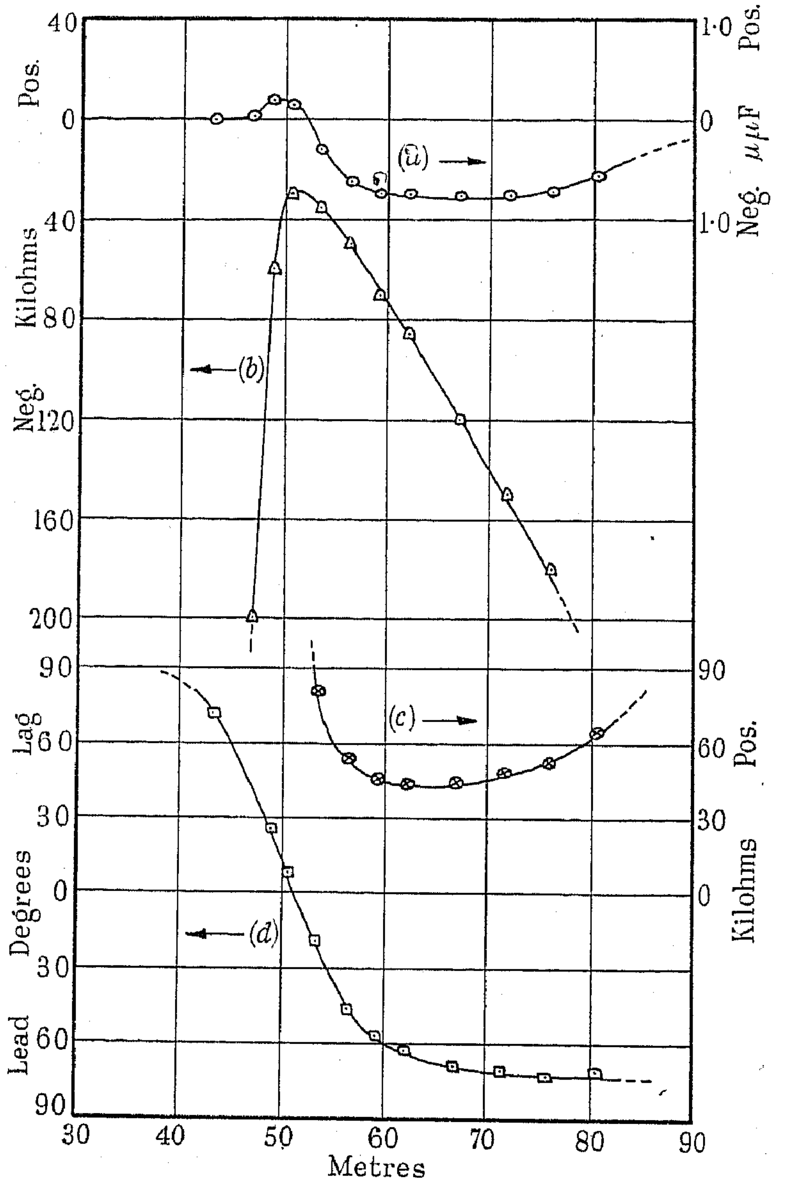

Fig. 8.—Variation of (*a*) change in valve capacitance, (*b*) negative resistance, (*c*) reactance, and (*d*) phase angle, with wavelength.

Anode voltage, 100 volts.
Filament emission, 10 mA.
Field strength, 1 790 gauss.
Amplitude, 13·6 volts (r.m.s.).

Capacitance —⊙——⊙—
Resistance —△——△—
Reactance —⊕——⊕—
Phase angle —⊡——⊡—

(e) Reactance and Phase Angle

It is known (J. S. McPetrie[6] and A. Giacomini[8]) that in the resonance regime the impedance of the magnetron has a reactive component, and this was measured in terms of the difference of the necessary tuning capacitance with the magnetron filament cold and hot. The apparent resistance and capacitance are shown as a function of wavelength in Fig. 8, from which it will be seen that the magnetron operation in general causes an apparent decrease of electrode capacitance. The corresponding curves of reactance and phase angle are also shown; it will be seen that the latter changes from near 90° lead to near 90° lag, and this is because $-R$ tends to infinity more rapidly than C (change in capacitance) tends to zero. Measurements show that C varies inversely as the amplitude, and hence the phase angle is independent of amplitude. The phase angle is also independent of wavelength provided this is accompanied by proportionate changes of field strength. Since $-R$ has been shown to vary as λ, it follows that c is sensibly independent of wavelength, its greatest value being of the order 1 $\mu\mu$F in this valve.

(f) Generation of Oscillations

For any assigned load resistance the power output can be calculated when the amplitude is known, and this output is found to reach a maximum as the field strength is increased. This maximum output is shown as a function of load resistance in Fig. 9(*a*) for five wavelengths, and it will be seen that each rises to a blunt maximum. The optimum value of field does not, however, change much over a wide range of load resistance. Determination of I_a makes it possible to calculate the efficiency at maximum output, and this is found to be of the order of 40 %–60 % and is sensibly independent of wavelength. It may be deduced from Fig. 4 that the efficiency in general is nearly independent of amplitude and resistance, but it is found to vary with anode voltage and filament emission. The maximum output is shown as a function of frequency in Fig. 9(*b*), and results in a linear relation. The optimum load decreases linearly with wavelength as in Fig. 5. The magnetic field at maximum output is shown in Fig. 10(*a*) as a function of wavelength and in Fig. 10(*b*) as a function of anode voltage, and these linear relations result in a value of K that differs little from a mean of 292. When generating oscillations in this regime under conditions where the input is large the magnetron shows the well-known phenomenon of additional heating of the filament. If under certain conditions a continuous-wave output is not essential the

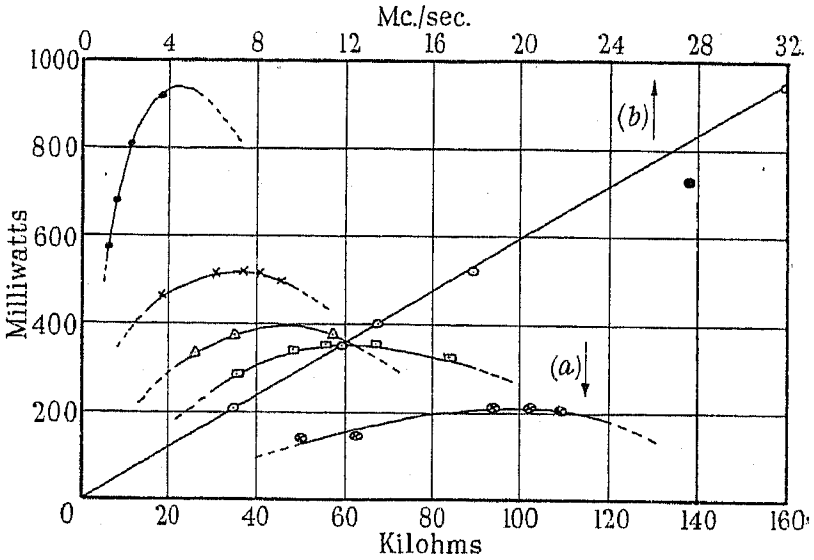

Fig. 9

(*a*) Variation of power with load resistance at various different frequencies.
(*b*) Variation of maximum power frequency.

Anode voltage, 200 volts.
Filament emission, 10 mA.
32·2 Mc./s., 9·27 m. —●——●—
17·8 Mc./s., 16·85 m. —×——×—
13·4 Mc./s., 22·35 m. —△——△—
11·2 Mc./s., 26·75 m. —⊡——⊡—
6·9 Mc./s., 43·30 m. —⊕——⊕—
Maximum power —⊙——⊙—

restrictions imposed by anode dissipation and filament heating can be overcome by applying the anode supply voltage in the form of sharp rectangular pulses, the voltage of which can be up to several times the normal value. An additional advantage is that the minimum wavelength is considerably reduced, as indicated previously, and using a small 4-segment valve rated at 150 watts, peak outputs of several kilowatts have been obtained on a wavelength of 25 cm.

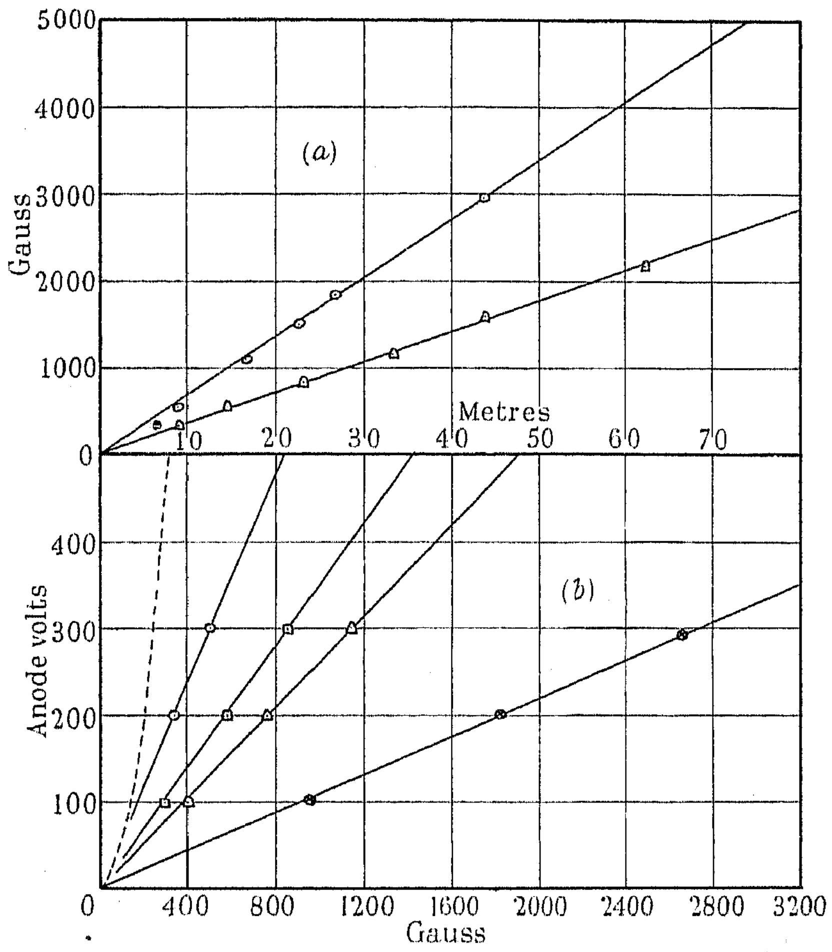

Fig. 10

(*a*) Variation of field strength for maximum power and for minimum wavelength, with wavelength.
Anode voltage, 200 volts. Maximum power —⊙——⊙—
Anode voltage, 100 volts. Minimum wavelength —△——△—

(*b*) Variation of anode voltage with field strength, for maximum power.
Cut-off condition - - - - - -
Wavelength 4·8 m. —⊙——⊙—
Wavelength 8·0 m. —⊡——⊡—
Wavelength 11·0 m. —△——△—
Wavelength 26·8 m. —⊕——⊕—

The relation between power (E^2/R) and I_e shown in Fig. 7(*a*) indicates that modulation by control of the space current (as by means of a suitable control grid) is possible provided the current is below the optimum value, but the linearity is not good and the mean output would be low. Fig. 3 shows that anode modulation is possible, both when the amplitude increases with V and when it decreases. In the former case the modulation depth is limited (and sometimes zero), and if this limit is exceeded a hysteresis effect occurs which at the longer wavelengths is sufficient to prevent the oscillations from building up again without circuit readjustment. Similar results would be expected from field-strength modulation, but it has been shown that if H and V are altered in the same ratio then the amplitude for a given resistance alters similarly: thus nearly full and linear modulation is possible in this manner. It is apparent from Fig. 4(*a*) that linear modulation is also possible by variation of the load resistance by a suitable modulating system, but the depth is restricted to an extent depending on the field.

The frequency stability as a property of the magnetron has been measured in the dynatron regime by the author,[1] who found that in the neighbourhood of the best operating conditions the stability is of the order 200 parts in 10^6. Small variations in valve resistance due to changes in the operating conditions have an effect on the generated frequency, but the curves of Fig. 2 suggest that the behaviour of the magnetron in the resonance regime in respect of frequency stability should be the same as that for any " dynatron type " generator. Frequency-changes due to small variations in the valve reactance can be reduced by the use of circuits with a small L/C ratio and low values of rejector resistance, but these conditions are not conducive to good circuit stability.

Consideration of Fig. 4(*a*) shows that self-oscillation is not possible when the load resistance is less than a certain minimum value which depends on H, but if a voltage of sufficient magnitude is applied the valve exhibits negative resistance at the wavelength of the applied potential. Thus the effective value of the load resistance to the source of fluctuating potential is reduced, resulting in amplification. For example, if the load resistance is made 60 kΩ with $H = 2\,260$ then for an amplitude of 40 volts (r.m.s.) the negative resistance is 78 kΩ, making the effective load resistance 260 kΩ. The driving power is 6·1 mW and the output is 26·7 mW, resulting in a power amplification of 6·5 db. The linear relation between negative resistance and amplitude shows that the valve would amplify a wave with a depth of modulation depending on the field strength, but the modulation wave would be distorted and, since the driving power varies during the cycle, it would be necessary for this stage to have a low internal resistance to avoid further distortion.

Fig. 4(*a*) also suggests that, for any given conditions, knowledge of the voltage amplitude when the valve and a given circuit act as a generator determines the rejector resistance of the circuit, and as a result of the linear relation between resistance and wavelength in the magnetron it is possible to calibrate it at long wavelengths and measure circuit resistances at much shorter wavelengths. Although the curves shown apply to symmetrical circuits similar ones could be derived for the case in which the valve is operated in an unsymmetrical manner.

(4) IMPEDANCE IN THE ELECTRONIC REGIME

(a) Filament Emission and Angle of Tilt

Since the properties are practically independent of any external impedance the valve was operated as a self-excited generator, the wavelength of the external oscillation being carefully measured. The circuit used was a pair of parallel wires several wavelengths long (λ about 30 cm.) kept away from other objects, the H.T. input being by means of a sliding bridge in the form of a large metal screen, and the idle portion of the line being terminated by a suitable resistor. The amplitude was usually indicated by a thermocouple and millivoltmeter placed near the valve, the position being adjusted for maximum millivoltmeter reading. The wavelength was measured by noting the bridge positions for maximum amplitude, and the mean of several half-wavelengths was taken. As the resistance in the electronic regime is particularly sensitive to changes in magnetic field the latter was accurately determined by using an electromagnet with a stalloy yoke so that hysteresis was reduced to a minimum, and by careful calibration with a fluxmeter. The presence of undesired long-wavelength oscillations, inherent to the form of circuit adopted, was eliminated by suitable adjustment of emission and tilt.

It is known that the amplitude depends considerably on the angle of tilt, and it was found that the optimum tilt varied with the filament emission in the manner shown in Fig. 11(*a*) for a given set of operating conditions. Thus for vanishingly small emissions the optimum tilt appears to be zero, although the valve resistance would then be very high. Thus tilt is necessary to reduce the space charge caused by the larger emissions, an effect which is in agreement with theory. It is likely that the advantage of slight tilt is the reduction in resistance previously discussed, although the fact that the magnetic field only just exceeds the cut-off value means that a large number of electrons collide with the anode, resulting in low efficiency. The amplitude increases to a maximum with emission and tilt and then falls for larger currents. Thus there appears to be some mechanism in the valve, as noted for the resonance oscillations, which increases the normal space charge, since saturation should not occur until the emission is of the order of several amperes with the high anode-voltages employed.

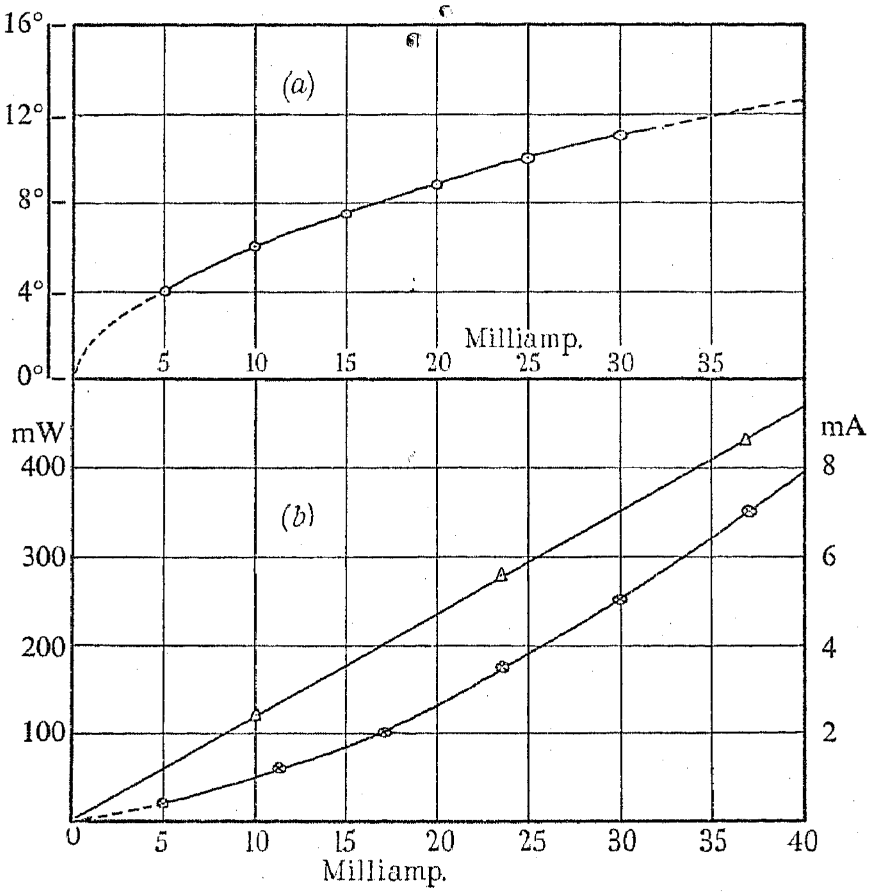

Fig. 11

(*a*) Variation of optimum angle of tilt with filament emission

(*b*) Variation of anode-current increase and of equivalent filament heating with anode current.

Anode voltage, 700 volts.	Optimum tilt	—○——○—
Field strength, 405 gauss.	Anode-current increase	—⊕——⊕—
Wavelength, 30 cm.	Filament heating	—△——△—

The determination of the valve resistance itself is difficult owing to the very short wavelengths and the finite size of electrodes, which make the valve equivalent to a small uncertain length of line. It was found that the amplitude indication was little affected by changing the line impedance from 250 to 500 ohms and that in order to obtain maximum brightness of a load lamp whose steady-current resistance was 10 ohms it was necessary to place it near a voltage antinode. Since the power outputs were of the order of 1 watt and the voltage amplitude appeared to be only a few volts it would seem that the resistance has a very low value (in the neighbourhood of 20 ohms) at this wavelength. Extrapolation of the resistance values found at the longer wavelengths in the resonance regime would indicate such a resistance, and it is probable that the magnetron resistance is always low near the cut-off condition. This would be explained by the fact that the electrons at the outer portion of their orbits pass very close to the segment gaps and experience the maximum force due to any potential differences.

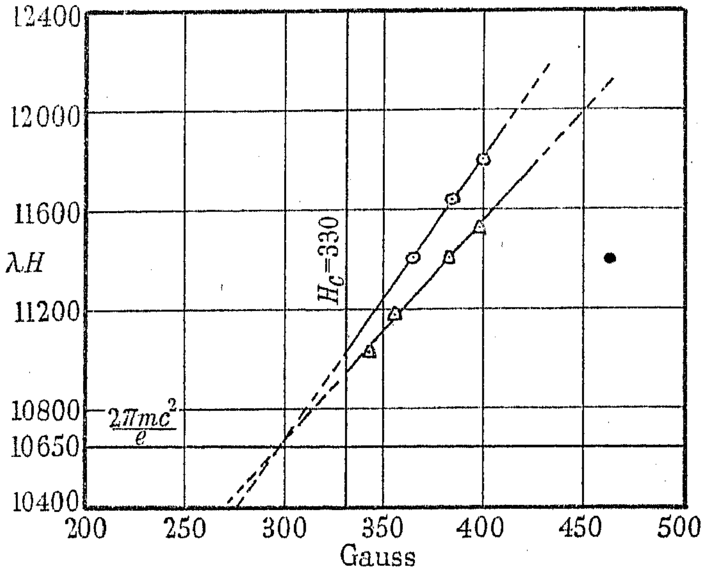

Fig. 12.—Variation of λH (in cm. gauss) with field strength.

Anode voltage, 600 volts.
Emission 27 mA, tilt 9·5° —○——○—
Emission 14 mA, tilt 8·0° —△——△—

(b) Wavelength and Magnetic Field

It is well known[9] that negative resistance in the electronic regime exists at wavelengths such that λH is a constant of about 11 000, and, moreover, that this value appears to be closely related to the transit time of the electrons.[10] The values of this product were measured under different conditions, and the variation of λH with H is shown in Fig. 12 to be sensibly linear, the required field-strengths being slightly greater than the cut-off value. Fig. 13 shows that λH increases with the emission but tends to a minimum when this becomes small. The corresponding curves of tilt (also shown) result in an effect to be expected from Fig. 11(*a*).

It will be observed that the minimum values to which λH tends are generally within the range 11 000–11 400, the lower values relating to conditions in which the space charge would be expected to be less. It is known that if the magnetic field is such that the electrons just miss the anode then when the cathode radius is made zero the electron transit-time tends to the value

$$T = \frac{2\pi mc}{He}$$

for vanishingly small emissions, and to the first order this time is not changed by small angles of tilt. A wave with this periodic time gives $\lambda H = 10\,650$, and for the cathode radius of the valve used graphical integration of A. W. Hull's equations gives a value of λH about 15 % greater.

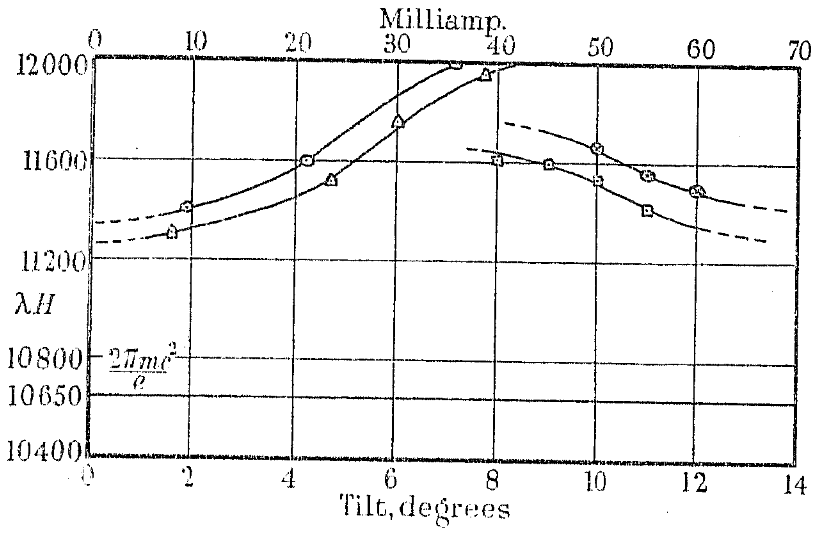

Fig. 13.—Variation of λH (in cm. gauss) with filament emission and with angle of tilt.

Field strength, 384 gauss.
Anode voltage 600 volts, tilt 8° —⊙——⊙—
Anode voltage 600 volts, tilt 10° —△——△—
Anode voltage 600 volts, emission 21 mA —⊡——⊡—
Anode voltage 630 volts, emission 36 mA —⊕——⊕—

F. B. Pidduck[11] shows that the effect of space charge in the cylindrical (and in the planar) magnetron is to increase the electron transit-time, and while the observed changes in λH can be qualitatively explained by changes in space charge it seems that the observed value is less than would be expected from theory. It is not possible to arrive at any definite conclusion since the oscillations generally occur at fields exceeding the cut-off value, and with small alternating voltages between the segments it is difficult to estimate the transit time under such conditions.

Table 2

λ	V	H	α	I_e	$\lambda V/H$	$\lambda V/(44H)$	λH	$\lambda H/10\,650$	$-R$
cm.	volts	gauss	degrees	mA					kΩ
370	200	1 120	17	6	66	1·5	415 000	38·9	Approx. 20
370	300	1 140	3·5	2	97	2·2	420 000	39·4	
370	400	1 120	17	6	132	3·0	415 000	38·9	
370	500	1 100	8	5	168	3·8	407 000	38·2	
370	300	371	0	1·5	297	6·8	137 000	12·8	
370	300	640	8	15	173	3·9	237 000	22·2	
370	300	1 680	16	20	66	1·5	620 000	58·2	

As in the case of resonance oscillations it is possible to calculate the values of $\lambda V/H$ over the range of negative resistance. In particular, the value for maximum output varies only slightly and has a mean of 44 which is about one-seventh the value of 292 for maximum output of resonance oscillations. This suggests that there may be other regimes of negative resistance with such integral relations, and some of these have been found. The resistances of the magnetron in these minor regimes tend to be high since oscillations are found only at comparatively high anode voltages, and in some cases large angles of tilt and critical values of filament emission are necessary. Some typical results are given in Table 2.

The value of $\lambda H/10\,650$ is usually known as the " order of oscillation," and is taken as unity for the normal electronic oscillation. The first four results show that oscillation can take place when, as in the electronic regime, λH tends to be constant, although the order of oscillation is very high. All the results show that intermediate values of $\lambda V/H$ are possible, but the fact that the optimum tilts and emissions are different makes it difficult to come to any definite conclusion regarding the relation between them. No extensive measurements were made of the properties of the valve under these conditions, but it was noted that they tended to be those associated with the resonance oscillations as well as the electronic type. It is important that these minor oscillations show properties similar to those of the other two regimes, since it points to a similarity between the latter. Thus many of the valve properties determined at longer wavelengths could be applied, at least qualitatively, to the electronic regime of negative resistance, and it is possible that they could be applied in part to electronic oscillations occurring in thermionic valves in general.

(c) Additional Filament Heating

As with resonance oscillations, additional filament heating occurs, which is revealed by the feed current exceeding the value for H zero. This phenomenon has been partly examined by O. Pfetscher and W. Puhlmann[12] and A. P. Maidanov.[13] Since I_a is only slightly less than I_e any observed increase in I_a is due mainly to an increase of emission. Fig. 11(*b*) shows this increase and the equivalent filament heating power (derived from the emission characteristic) as a function of anode current for a constant anode voltage. This effect increases with the input to the valve, and since the output falls appreciably for large emissions it is evident that this effect can occur when there is no external evidence of oscillation.

The filament was heated from a sensibly constant-current source and the changes in battery filament heating due to resistance variations were determined by a potentiometer and millivoltmeter. After the equivalent heating has been corrected for this some energy remains, namely that imparted to the filament during operation and which must come from the anode supply. Moreover, the filament-resistance changes correspond to those which would occur if the increased anode current were entirely due to an increase in filament emission; and thus in this case any effects due to secondary emission, positive-ion current, or other causes are negligible.

This energy is expressed in Fig. 14(*a*) as a function of changes in length of the external circuit. It will be noted that this effect is greater when the external output is a minimum (and actually nearly zero), and suggests that the mechanism causing this heating would be present with the anode segments short-circuited. The rises in anode current in this experiment were of the order shown in Fig. 11(*b*). Fig. 14(*b*) shows this heating energy as a function of angle of tilt, and some results for the larger resonance regime heating are also included. It would seem that this heating is due to the electrons returning to the filament with higher velocities than those with which they left, since it can be demonstrated[4] in other ways that it is possible for the electrons in the course of their orbits to have greater velocities than those given by simple calculation. It is uncertain, however, whether it is a necessary condition that oscillation shall be present.

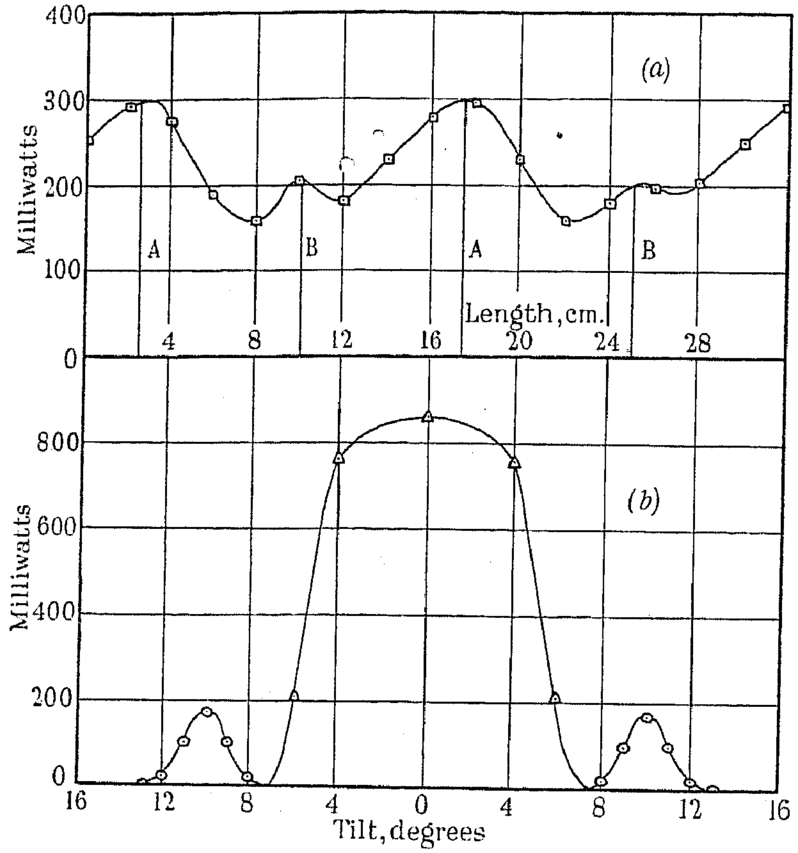

Fig. 14

(*a*) Variation of filament heating with change in length of external circuit.
(*b*) Variation of filament heating with angle of tilt.

Anode voltage, 700 volts.
Field strength, 381 gauss.
Filament emission, 35 mA.
" A " represents minimum external output.
" B " represents maximum external output.
Wavelength 30 cm., tilt 10° —□——□—
Wavelength 30 cm. —○——○—
Wavelength 350 cm. —△——△—

(5) CONCLUSIONS

A description has been given of experiments made to measure the impedance of the magnetron and to investigate its special properties in the three regimes of oscillation which are usually recognized as distinct. Many direct properties of the valve have been found and these have been discussed, and where possible agreement or conflict with the known theory of the magnetron has been pointed out. There appears to be experimental evidence which strongly suggests that these regimes are not entirely distinct, and that the valve properties may differ in them only because of the varying effects due to electron motion or allied phenomena.

(6) ACKNOWLEDGMENTS

The work described in the paper was carried out in the Engineering Laboratory of the University of Oxford, and the author is indebted to Mr. E. B. Moullin, M.A., Member, for his advice and interest in the experiments and for help in the preparation of the paper.

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DISCUSSION ON
"QUASI-STABLE FREQUENCY-DIVIDING CIRCUITS"*

Mr. H. Mahmoud (Egypt) (*communicated*): In working out some of the properties of the "synchronized oscillator," I found it possible to calculate the amplitude of oscillation as well as the range of synchronization by the simple means of selecting the proper terms from the expression for the anode current. The same method could be applied when the system is used for frequency division, and I think the result of such a treatment will be a useful addition to the paper.

Fig. A shows a triode having a tuned circuit in series with the synchronizing source in the anode circuit, and reaction applied to the grid circuit. The losses in the circuit are represented by a shunt arm of conductance G in order to simplify the analysis, and the results obtained apply as well to the more usual case when the losses are associated with the inductance.

Let us assume that the resonant circuit is tuned to a frequency $p/(2\pi)$, while the source has a frequency $q/(2\pi)$, where p and q are nearly equal.

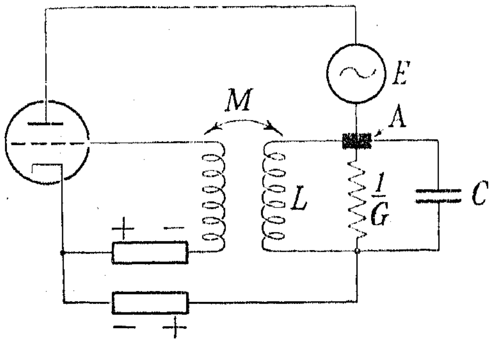

Fig. A

Under operating conditions let the synchronizing source be represented by

$$E' = E \sin (2qt - \theta)$$

and the voltage across the tuned circuit by

$$v = V \sin \omega t$$

Since the tuned circuit discriminates strongly against frequencies far from its resonant frequency, then ω will be nearly equal to p.

The alternating component of the anode current can be approximately represented by

$$i_a = a(v_a + \mu v_g) + b(v_a + \mu v_g)^2 + c(v_a + \mu v_g)^3 \quad (1)$$

From Fig. A we find the value of $(v_a + \mu v_g)$:—

$$v_a + \mu v_g = (1 - \mu M/L) V \sin \omega t + E \sin (2qt - \theta)$$

and by substituting this value in equation (1), and selecting only those terms with angular frequencies near p, we get for the alternating component of the anode current:—

$$i_a = akV \sin \omega t + \tfrac{3}{4}ck^3V^3 \sin \omega t + 2\,.\,\tfrac{3}{4}ckE^2V \sin \omega t + bkVE \cos \left[(2q - \omega)t - \theta\right] \quad . \quad . \quad (2)$$

where $k = (1 - \mu M/L)$.

By applying Kirchhoff's law for the currents meeting at point A we get

$$- i_a = GV \sin \omega t + CV \cos \omega t - (1/\omega L) V \cos \omega t \quad (3)$$

From equations (2) and (3) it may be concluded that $(2q - \omega)$ must equal ω, or in other words the frequency of the voltage across the tuned circuit is exactly half that of the impressed voltage.

Using this result and equating terms of $\cos \omega t$ and $\sin \omega t$ separately, we find

$$VbkE \cos \theta = (V/\omega L) - \omega CV \quad . \quad . \quad . \quad (4)$$

and

$$Vk(a + \tfrac{3}{4}ck^2V^2 + 2\,.\,\tfrac{3}{4}cE^2 + bE \sin \theta) + VG = 0 \quad (5)$$

By putting $E = 0$ in equation (5) we get the case of an ordinary regenerative amplifier which will start to oscillate as soon as

$$ka + G \leq 0 \quad \text{i.e. } a(\mu M/L - 1) \geq G \quad \text{or } \mu M/L \geq 1 + G/a\dagger$$

If the system is to remain stable in the absence of the synchronizing e.m.f., then the coupling should be smaller than the value indicated above, i.e.

$$\mu M/L = 1 + G/an \quad \text{(where } n \text{ is greater than unity)}$$

or

$$- G = an(1 - \mu M/L) = ank$$

Putting this value of G in equation (5), then

$$bE \sin \theta + \tfrac{3}{4}c(k^2V^2 + 2E^2) = a(n - 1) \quad . \quad (6)$$

The factor c in all the above equations is negative for all practical cases, and therefore we put $\frac{3}{4}c = - \gamma$. Equations (4) and (6) then become

$$bE \cos \theta = \left(\frac{an}{G}\right)(\omega C - 1/\omega L) = \left(\frac{an}{G}\right)(Z/\omega L) = \beta Z \quad (7)$$

$$bE \sin \theta - \gamma(k^2V^2 + 2E^2) = a(n - 1) = \alpha \quad . \quad (8)$$

where

$$Z \simeq 2\frac{\omega - \omega_0}{\omega_0}, \ \omega_0^2 = \frac{1}{C}, \ \beta = \frac{an}{G\omega L}, \text{ and } \alpha = a(n - 1)$$

Equations (7) and (8) give the complete behaviour of the system. Equation (7) gives the variation of the phase angle θ (between the impressed e.m.f. and a fictitious e.m.f. in phase with the output voltage but of double the frequency) when the frequency of the impressed e.m.f. is varied around a value equal to double the resonant frequency of the tuned circuit.

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* Paper by R. L. Fortescue (see *Proceedings of the Wireless Section*, 1939, **14**, 171). Reprinted from *Journal I.E.E.*, 1939, **85**, p. 646.

† It will be seen that the factor $1/a$ is the a.c. anode resistance of the valve when working under linear conditions.

Equation (8) when rearranged gives the amplitude of the output voltage:—

$$V^2 = (1/\gamma k^2)[bE \sin\theta - 2\gamma E^2 - \alpha] \quad . \quad . \quad (9)$$

Therefore we see that, in general, the output will vanish unless the amplitude E lies between two values given by the roots of equation (9) when V is made zero.

We also note that if the amplitude E is held at a favourable value and the frequency is slowly varied, a point will be reached which makes $\sin\theta$ of such a value that the amplitude of the output is reduced to zero. The range over which the phase angle θ varies is thus seen to be between $\frac{1}{2}\pi$, a value less than π, and a value greater than 0.

The range over which the system will operate is then found by eliminating the angle θ from equations (7) and (8) and putting $V = 0$ in the latter. Then

$$b^2E^2 = \beta^2Z^2 + [\alpha + 2\gamma E^2]^2 \quad . \quad . \quad . \quad (10)$$

This is the equation of an ellipse and of the form of the experimental curve given in Fig. 4 (page 175 of the paper).

Had the synchronizing e.m.f. been introduced in the grid circuit instead of in the anode circuit, then equations (1) to (10) would have had to be modified by replacing E by μE.

The case considered above is for division by 2, but the method outlined applies equally well for other cases.

An alternative circuit for frequency division is shown in Fig. B. The circuit is arranged to be initially unstable, and the extra reaction is cut out as soon as an output of the required frequency is obtained.

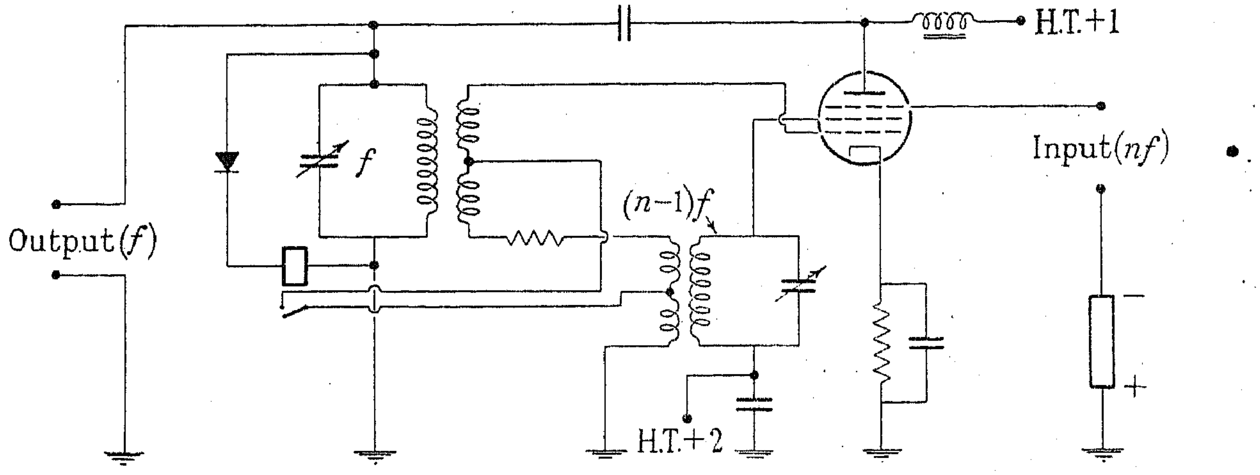

Fig. B

Mr. R. L. Fortescue (*in reply*): Mr. Mahmoud gives a steady-state analysis of a simple oscillator operating as a frequency divider and shows that quasi-stable working should be possible if direct reaction is reduced, by loosening grid-anode coupling, to a point just below the minimum required for self-oscillation. It seems, however, that this condition is difficult to obtain in practice, such an oscillator usually continuing to oscillate, once started, whether synchronizing drive is present or not; though it is easy to adjust it so that it will not *start* free oscillation without this drive. The reason for this behaviour is probably the form of the valve characteristic of the commoner valves, which either includes appreciable terms higher than the second order in v or may have a ratio $b^2/(ac)$, governing the intermodulation/direct reaction ratio, so small that the permissible range of grid coupling for quasi-stable operation is too critical to be maintained without special precautions. Even in the absence of such effects, however, the steady-state analysis may be misleading. Mr. Mahmoud assumes the voltage across the tuned circuit to be of the form $v = V \sin\omega t$, when $\omega \simeq p$. Actually such an approximation is only valid if $\omega = p$, for if the oscillator is not synchronized the voltage takes a complex form best expressed by a modulated wave $v = Vf(t) \sin pt$, there being, speaking loosely, a beat between forced and free oscillations. The steady analysis thus only gives conditions which must be satisfied if a continuous state of frequency division exists. Such conditions are necessary, but not sufficient, for the assumed equilibrium state may be an unstable one. A steady-state solution for a normal valve oscillator is obtained by putting all a.c. voltages equal to zero; but it is unstable until the reaction is reduced below a certain level. By means of isoclines it can be shown that with a simple free oscillator stable and unstable states alternate with a.c. amplitude; i.e. if the origin, where all a.c. voltages are zero, is unstable, the next steady state is stable, followed by another state with higher a.c. voltages satisfying the analysis but unstable, etc. Probably this is also true in the case of driven oscillators and frequency dividers, and a quasi-stable system would then have a stable origin and stable oscillating state separated by an unstable equilibrium of lower a.c. amplitude. To ensure that the divider settles to the stable oscillation and does not return to the stable origin, with zero a.c. output at divided frequency, the special starting system is required. It will be interesting to see whether the future produces an analytic method similar to the isocline system which can be applied to this problem of the unsynchronized divider to settle these points.